

DETERMINATION THE NUMBER OF ANTS USED IN ACO ALGORITHM VIA GRILLAGE OPTIMIZATION

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Abstract: Ant colony optimization (ACO) algorithm is one of the artificial intelligence methods used in structural optimization. Values of some optimization parameters must be determined before the optimization process in most of the artificial intelligence based optimization algorithms. Determination of the values of these optimization parameters is essential especially for the time required for the optimization process and the quality of results achieved. Pheromone update coefficient, number of ants in the colony, number of depositing ants, penalty coefficient are the main optimization parameters in ACO algorithm. This study is focused on the number of ants in the ant colony. This research is realized using the optimization of grillage structure which is one of the well-known optimization problems in the literature. Minimization of the weight of structure is the objective function of the optimization problem, and the member sizes of grillages are considered as discrete design variables. Displacement and strength restrictions are considered as constraints according to manual of LRFD-AISC. A computer program is coded in BASIC to accomplish the structural design and optimization procedures. Numerical examples from literature are optimized using different number of ants to determine the effect of the number of ants on the optimization process. At the end of the study, some inferences are presented on the number of ants to be used in the colony.

Keywords: Ant colony optimization, Structural optimization, Number of ants, Grillage structure

Izgara Sistemlerin Optimizasyonu Üzerinden Karınca Koloni Optimizasyon Algoritmasında Karınca Sayısının Belirlenmesi

Öz: Karınca koloni optimizasyon algoritması, yapısal optimizasyonda kullanılan yapay zekaya dayalı yöntemlerden biridir. Yapay zekaya dayalı optimizasyon algoritmalarının çoğunda bazı optimizasyon parametrelerinin değerleri optimizasyon sürecinin öncesinde belirlenmesi gerekmektedir. Bu optimizasyon parametrelerinin değerlerinin belirlenmesi özellikle optimizasyonun işlemi için gerekli süre ve ulaşılan sonuçların niteliği açısından önemlidir. Feromon güncelleme katsayısı, kolonideki karınca sayısı, feromon bırakacak karınca sayısı, ceza katsayısı karınca koloni algoritmasındaki başlıca optimizasyon parametreleridir. Bu çalışma ise kolonideki karınca sayısına odaklanmaktadır. Bu araştırma, literatürde sıkça ele alınan optimizasyon problemlerinden biri olan, ızgara sistemlerin optimizasyonu üzerinden gerçekleştirilmiştir. Yapı ağırlığının minimum değerinin belirlenmesi optimizasyon probleminin amaç fonksiyonu ve ızgara sisteminin oluşturan elemanların enkesit ebatları ise ayrık tasarım değişkenleri olarak dikkate alınmıştır. Yerdeğiştirme ve dayanım limitleri "LRFD-AISC" yönetmeliğine göre sınırlayıcılar olarak alınmıştır. Yapısal tasarım ve optimizasyon süreci için gerekli işlemleri yapmak üzere "BASIC" dilinde bir bilgisayar programı kodlanmıştır. Karınca sayısının optimizasyon süreci üzerindeki etkisini belirlemek için literatürden seçilen sayısal örnekler farklı karınca sayıları kullanılarak optimize edilmiştir. Çalışmanın sonucunda, kolonide kullanılması gereken karınca sayısına ilişkin bazı çıkarımlar sunulmuştur.

Anahtar Kelimeler: Karınca koloni optimizasyonu, Yapısal optimizasyon, Karınca sayısı, Izgara yapı

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1. INTRODUCTION

Structural optimization problems are one of the important application areas for the artificial intelligence based optimization algorithms in the academic literature. Many different artificial intelligence based algorithms, e.g. genetic algorithm, simulated annealing, particle swarm, harmony search, cuckoo search, artificial bee colony, teaching-learning based algorithm, firefly algorithm etc., have been used for optimization of structural optimization problems since 1990s (Daloğlu et al., 2017; Kaveh et al., 2017; Aydoğdu et al., 2016; Moezi et al., 2015; Mashayekhi et al., 2016; Tort et al., 2017; Farshchin et al., 2016; Çarbaş, 2016; Çarbaş et al., 2013). Another one of these algorithms is the ACO algorithm which is used in this study. ACO algorithms mimic the ability of ant colonies to find the shortest path between food source and nest (Dorigo, 1992).

Generally, artificial intelligence based optimization algorithms need an iterative search process to reach optimum results. Each of these algorithms have some optimization parameters; and, values of these parameters must be determined carefully to reach the best results as soon as possible. In an ACO algorithm, pheromone update coefficient, number of ants in colony, number of depositing ants and penalty coefficient are the main optimization parameters. This study focuses on the number of ants in the colony.

Grillages are selected as structural optimization problem in the study. Cross-sectional sizes of girders are considered as discrete design variables; and, a list of W-sections is predetermined for possible values. Displacement, flexural and shear strength are constrained according to LRFD-AISC Manual of Steel Construction (1999). Weight of the structure is considered as objective function of the optimization problem.

Artificial intelligence based algorithms was used in optimization of grillages previously, e.g. Saka et al. (2000) used genetic algorithm (GA), Saka and Erdal (2009) used a harmony search based optimization (HSBO) algorithm, Kaveh and Talatahari (2010) used the charged system search (CSS) algorithm, Kaveh and Talatahari (2012) used a hybrid combining charged system search and particle swarm optimization (PSO) algorithm, and Dede (2013) used teaching learning based optimization (TLBO) algorithm.

A computer program is coded in Basic to accomplish the necessary calculations for the optimization and design procedures. A numerical example from the literature is optimized several time considering different member grouping using this computer program.

A simplified ant colony optimization (SACO) algorithm which uses a simpler formulation than those of in the literature is used in this study (Aydın and Yılmaz, 2014; Aydın, 2016). The purpose of this study is to determine how the number of ants affects the optimization process. For this purpose, structural system selected is optimized using different ant colonies which have different number of ants. Consequently, relation among the number of ants, the quality of the results and the number of iteration is researched.

A similar study was realized for the determination of effective number of depositing ants by Aydın (2016); and it was concluded that the better results were reached in the case of using lesser number of depositing ants. In that study, it was also recommended the use of elitist approach in which only the best ant deposit pheromone. Accordingly, it is supposed in in this study that only the best ant deposits pheromone.

2. STRUCTURAL OPTIMIZATION PROBLEM

2.1. Objective Function

In optimization of steel structures, weight of the structure is generally selected as optimality criterion instead of the structural cost. Therefore, the aim is to find out the minimum-weighted structure in optimization of a steel grillage structure, and objective function (W) of the optimization problem can be formulated as

$$W = \sum_{i=1}^{nm} G_i l_i \tag{1}$$

where, nm is the number of members in the grillage structure, G_i is the unit weight of the member i and l_i is the length of the member i .

In the equation given above, the number of member and the length of member are design parameters of the structure and values of them do not change during the optimization process. Design variables are cross-sectional size of the members which is represented by the unit weight of member in equation (1). A discrete optimization is realized in this study; and, W sections list of LRFD-AISC Manual of Steel Construction (1999) is considered for the values of the design variables. Therefore, determination of the minimum-weighted structure means determination of the suitable values for the unit weight of the members from the considered list.

2.2. Penalized Objective Function

There is no doubt that minimum-weighted structure is constituted by using the minimum values of design variables; but, on the other hand, the structure must satisfy the constraints. So, in fact, the aim of the structural optimization is to find out the structure which do not violate the constraints. Accordingly, the objective function must be transformed to a penalized form depending on the violation of the constraints. A penalized objective function (Φ) is calculated for this transformation using the technique of Rajeev and Krishnamoorthy (1992) as

$$\Phi = W \cdot [1 + K \cdot P] \tag{2}$$

where K is the penalty coefficient which is used to determine how the constraints affect the penalized objective function, P is the penalty function which is calculated according to violation of constraints. In the general form, penalty function can be formulated as

$$P = \sum_{i=1}^{nc} p_i \tag{3}$$

where nc is the number of constraints, p_i is the penalty violation factor of the constraint i and it is determined in normalized form with the equation given below.

$$\left. \begin{aligned} p_i &= \frac{g_i}{g_{u,i}} - 1 && \text{if } g_i > g_{u,i} \\ p_i &= 0 && \text{if } g_i \leq g_{u,i} \end{aligned} \right\} \tag{4}$$

In this equation, g_i and $g_{u,i}$ are the calculated value and restriction for the constraint i , respectively.

2.3. Constraints

Strength (for flexure and shear) according to LRFD-AISC Manual of Steel Construction (1999) and displacement constraints are considered in this study as explained below.

2.3.1. Flexural Strength Constraint

Flexural strength constraint is expressed in the accordance with the regulations under consideration as given below.

$$M_{u,i} \leq \phi M_{n,i} \quad i = 1, 2, \dots, nm \quad (5)$$

In this control for the member i , $M_{u,i}$ is the factored service load moment and $\phi M_{n,i}$ is the flexural design strength where ϕ is the resistance factor given as 0.9 for flexure, M_n is the nominal flexural strength which is calculated according to AISC-LRFD (1999) for laterally supported rolled beams depending on the slenderness (λ) as

$$M_n = \begin{cases} M_p = F_y Z_x \leq 1.5 F_y S_x & \text{if } \lambda \leq \lambda_p \\ M_p - (M_p - M_r) \frac{\lambda - \lambda_p}{\lambda_r - \lambda_p} & \text{if } \lambda_p < \lambda \leq \lambda_r \\ M_{cr} = F_{cr} S_x & \text{if } \lambda > \lambda_r \end{cases} \quad (6)$$

where M_p is the plastic moment, F_y is the yield strength of the material, Z_x is the plastic modulus, S_x is the section modulus, F_{cr} is critical stress given as $0.69E/\lambda^2$, M_{cr} is the buckling moment and M_r is calculated as

$$M_r = \begin{cases} F_L S_x & \text{for buckling of flange} \\ R_e F_{yf} S_x & \text{for buckling of web} \end{cases} \quad (7)$$

in which

$$F_L = \min \begin{cases} F_{yf} - F_r \\ F_{yw} \end{cases} \quad (8)$$

In equations (7) and (8), F_r is the compressive residual stress in flange given as 69 MPa; F_{yf} and F_{yw} are the yield strength of flange and web, respectively; R_e is the hybrid girder factor given as 1.0 for non-hybrid girders. In equation (6), the values of λ , λ_p and λ_r are calculated for compression flange and web, respectively, as

$$\left. \begin{aligned} \lambda &= \frac{b_f}{2t_f} \\ \lambda_p &= 0.38 \sqrt{\frac{E}{F_y}} \\ \lambda_r &= 0.83 \sqrt{\frac{E}{F_L}} \end{aligned} \right\} \text{for compression flange} \quad (9)$$

and

$$\left. \begin{aligned} \lambda &= \frac{h}{t_w} \\ \lambda_p &= 3.76 \sqrt{\frac{E}{F_y}} \\ \lambda_r &= 5.70 \sqrt{\frac{E}{F_y}} \end{aligned} \right\} \text{for web} \quad (10)$$

where b_f is the width of flange, t_f and t_w are the thickness of flange and web, respectively, h is clear height of the web (excluding fillets) and E is the young modulus of the material.

In this study, it is considered that grillages are constituted by hot rolled W sections; therefore, web local buckling is not considered as a constraint. The nominal flexural strength must be less than or equal to plastic moment of the cross section for all three cases of the slenderness ratio.

2.3.2. Shear Strength Constraint

Shear strength constraint to the regarded regulation is also expressed as

$$V_{u,i} \leq \phi V_{n,i} \quad i = 1, 2, \dots, nm \quad (11)$$

where for the member i , $V_{u,i}$ is the shear force according to the factored service load and $\phi V_{n,i}$ is the shear design strengths where ϕ is the resistance factor given as 0.9 for shear, V_n is the nominal shear strength which is calculated according to AISC-LRFD (1999) for rolled beams as

$$V_n = \begin{cases} 0.6 f_{yw} A_w & \text{if } h/t_w \leq 2.45 \sqrt{E/F_{yw}} \\ 0.6 f_{yx} A_w (2.45 \sqrt{E/F_{yw}}) & \text{if } 2.45 \sqrt{E/F_{yw}} < h/t_w \leq 3.07 \sqrt{E/F_{yw}} \\ A_w (4.52E)/(h/t_w)^2 & \text{if } 3.07 \sqrt{E/F_{yw}} < h/t_w < 260 \end{cases} \quad (12)$$

where A_w is cross sectional area of the web.

2.3.3. Displacement Constraint

In this study, maximum vertical displacements of some points in the grillage are constrained with the equation as given below.

$$\delta_i \leq \delta_{a,i} \quad i = 1, 2, \dots, ncp \quad (13)$$

where δ_i and $\delta_{a,i}$ are the calculated and the allowable displacement of joint i , respectively; ncp is the number of points whose displacements is restricted.

3. OPTIMIZATION OF THE STRUCTURE USING SACO

Ant colonies need new food sources to survive; accordingly, the principle duty of an ant in the colony is to find new food sources and to carry the foods to the nest. In natural habitat, there are generally more than one possible route between the food source and the nest. Ant colonies

must find out the shortest one among the probable routes for efficiency. Ants overcome this difficult task skillfully using a chemical material named as pheromone which is leaved on the route between the food source and the nest by ants. Amount of pheromone in a shorter route is more than a longer one because of evaporation. A new ant from the nest will follow the previous pheromones with a strong possibility, which means the following of the shorter route. Therefore, the shortest route will be find out by ant colony after a duration.

The ability of ants to find the shortest route between the food source and the nest was simulated by Dorigo (1992) to constitute a new optimization method named as ACO at the beginning of nineties. After that, ACO is used for different optimization problems one of which is structural optimization. Different versions of ACO are also used in the literature (Camp and Bichon, 2004; Hasançebi et al., 2011; Aydoğdu and Saka, 2012). The SACO algorithm preferred in this study is used previously by Aydın and Yılmaz (2014) and Aydın (2016).

In a discrete optimization problem, probable values of design variables are determined before the optimization process. These probable values are similar to probable routes in natural habitat of ants; accordingly, ants in the colony are represented by probable solutions of the optimization problem. Adaptation of natural process of ant colonies to a discrete optimization problem is illustrated in Figure 1. The example problem in the figure have two design variable which have four and three probable values, respectively.

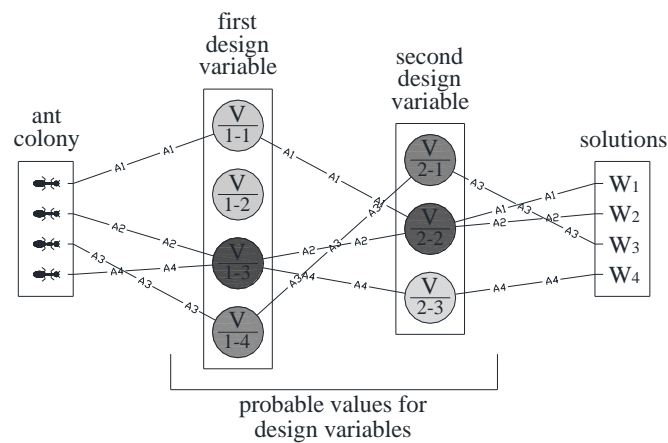


Figure 1:

Adaptation of natural process of ant colonies to a discrete optimization problem

It can be seen from Fig. 1 that each ant in the colony has a route to objective function; and, station of these routes are the probable values of design variables. Amounts of the pheromone on the probable values are represented by colors; accordingly, darker color demonstrate more pheromone. In SACO, amounts of pheromone are the selection probability of related values, and total amount of pheromone on the probable values of any design variable is equal to 1 (100%). It is supposed that there is equal amount of pheromone on each probable value of any design variable initially, and it is calculated as

$$Ph_{ij}^0 = \frac{1}{nv_i} \tag{14}$$

where Ph_{ij}^0 is initial amount of pheromone on the j^{th} probable value of i^{th} design variable; nv_i is the number of probable values for i^{th} design variable.

In this study elitist approach is considered as mentioned before; it means that only the best ant in the colony leaves pheromone on its route. Therefore, the amounts of the pheromone on

probable values used by the best ant are increased while the others are reduced. Reducing of pheromones on some probable values is similar to the evaporation in natural ant colony process. These modifications in pheromone amounts are named as pheromone update process which is formulated as given below.

$$\left. \begin{aligned} Ph_{ij}^k &= Ph_{ij}^{k-1} \cdot \left(1 - \frac{F \cdot nv_i}{nv_i - 1} \right) \rightarrow \text{for the best ant} \\ Ph_{ij}^k &= \frac{1 - Ph_{ij}^{k-1}}{Ph_{ij}^{k-1}} \cdot \frac{F \cdot nv_i}{nv_i - 1} \rightarrow \text{for the other ants} \end{aligned} \right\} \quad (15)$$

where Ph_{ij}^k is the amount of pheromone on j^{th} probable value of i^{th} design variable at the iteration k ; F is the pheromone update coefficient which determines the increment percentage of pheromones. Optimization process continues till the amount of pheromone in any value for each design variable reaches to the predetermined percentage.

4. NUMERICAL EXAMPLE

A 40-member grillage is selected as the numerical example from the literature to determine the suitable number of ant in the SACO algorithm. Dimensions, restraints and the loading condition of the selected grillage is shown in Fig. 2 where $q=200$ kN. Material properties are taken as: yield stress is 250 MPa, modulus of elasticity is 205 kN/mm², and shear modulus is 81 kN/mm². Total 272 W sections from W100x19.3 to W1100x499 from the list of LRFD-AISC Manual of Steel Construction (1999) are considered for probable values of design variables. Vertical displacements of 4 points in the center of grillage are restricted as the maximum 25 mm.

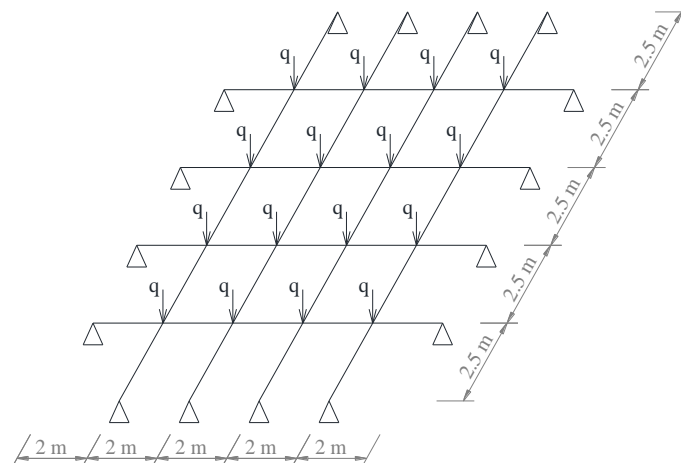


Figure 2:
40-member grillage structure

Different grouping approaches are used in this study to consider the effect of number of design variables. Members of grillage are collected in two, four and twelve groups in the first (grouping a), the second (grouping b) and the third (grouping c) approach, respectively, as illustrated in Fig. 3.

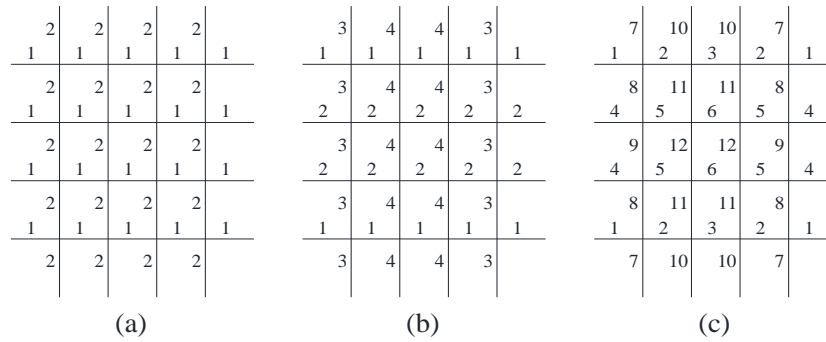


Figure 3:
Member grouping for (a) the first, (b) the second and (c) the third approach

For each of the three grouping approaches, grillage structure is optimized using colonies with 5, 10, 20, 40, 80, 160, 320 and 640 number of ants. The other optimization parameters are considered for all of optimization realized as

- Penalty coefficient (K): 0.1 ~ 0.5
- Pheromone update coefficient (F): 0.02
- Conversion percentage: 50%
- Maximum number of iterations: 500

Results of the optimization process are given in Table 1, Table2 and Table 3 for the first, the second and the third grouping approach, respectively.

Table 1. Optimum results for the first grouping approach (grouping a)

Number of ants	5	10	20	40	80	160	320	640
Group 1	W760x220	W840x176	W840x176	W840x176	W840x176	W840x176	W840x176	W840x176
Group 2	W200x15	W100x19.3	W200x15	W200x15	W150x13.5	W150x13.5	W150x13.5	W150x13.5
Iteration	163	75	102	74	47	54	45	38
Weight (kg)	9572	8002	7782	7782	7712	7712	7712	7712

Table 2. Optimum results for the second grouping approach (grouping b)

Number of ants	5	10	20	40	80	160	320	640
Group 1	W610x92	W360x51	W250x17.9	W410x46.1	W360x44	W310x38.7	W200x15	W410x46.1
Group 2	W920x201	W1000x222	W1000x222	W920x223	W1000x222	W1000x222	W1000x222	W1000x222
Group 3	W360x44	W200x15	W360x44	W150x18	W200x15	W310x21	W410x46.1	W150x13.5
Group 4	W310x67	W530x66	W410x67	W530x66	W530x66	W460x68	W460x52	W530x66
Iteration	317	252	164	163	141	123	69	87
Weight (kg)	8016	7476	7605	7463	7360	7453	7198	7353

It is shown in Table 1, Table 2 and Table 3 that the best weights are obtained using 80, 320 and 640 ants for the first, the second and the third grouping approach, respectively. All of the results of three approaches are collected in a graph in Fig. 4 to clarify how the number of ants affect the optimization process. Variations of the number of iterations and the number of analysis versus the number of ants are illustrated in Fig. 5 and Fig. 6 for all of three grouping approaches.

Table 3. Optimum results for the third grouping approach (grouping c)

Number of ants	5	10	20	40	80	160	320	640
Group 1	W360x64	W460x52	W310x52	W410x38.8	W410x38.8	W460x52	W310x23.8	W410X46.1
Group 2	W410x67	W530x72	W360x51	W360x51	W360x51	W410x38.8	W310x21	W460X52
Group 3	W410x38.8	W410x38.8	W250x28.4	W360x32.9	W150x18	W200x22.5	W200x15	W200X15
Group 4	W760x161	W690x152	W690x152	W690x170	W840x176	W840x193	W760x173	W760X147
Group 5	W920x201	W840x251	W1000x222	W1000x222	W920x201	W920x201	W920x201	W920X223
Group 6	W920x271	W840x251	W920x223	W1000x249	W1000x249	W920x238	W1000x272	W1000x222
Group 7	W310x23.8	W360x39	W250x17.9	W310x21	W360x32.9	W250x28.4	W460x52	W250x17.9
Group 8	W360x39	W310x32.7	W150x18	W310x21	W310x28.3	W310x28.3	W360x51	W250X22.3
Group 9	W360x57.8	W200x26.6	W100x19.3	W250x22.3	W310x23.8	W250x17.9	W250x22.3	W310X21
Group 10	W360x91	W460x68	W410x85	W530x74	W460x68	W610x82	W410x60	W530X74
Group 11	W610x92	W530x82	W460x74	W460x82	W530x66	W460x74	W410x60	W530X72
Group 12	W360x39	W310x21	W150x18	W150x22.5	W150x29.8	W150x22.5	W250x17.9	W310X21
Iteration	461	475	339	306	271	238	235	203
Weight (kg)	8119	7830	6967	7198	7027	7271	6925	6777

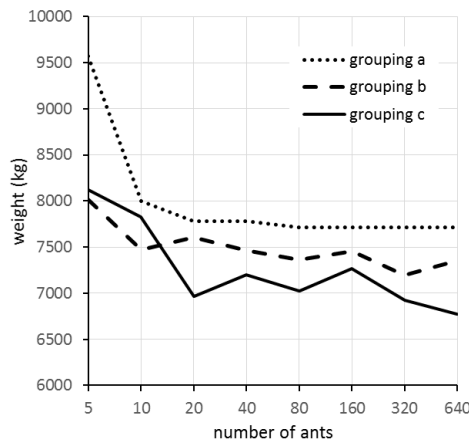


Figure 4:
Variation of the weight versus the number of ants

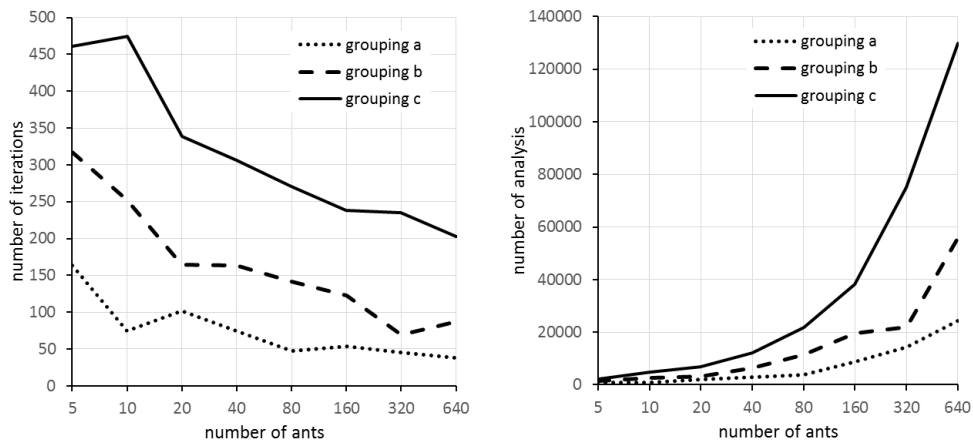


Figure 5:
Variation of the number of iteration and analysis versus the number of ants

From Fig. 5, the lesser iteration is needed in the case of using more ants. On the other hand, from Fig. 6, number of analysis and the time needed for the optimization process are getting higher in the case of using more ants.

The example with the second grouping approach is previously handled using different optimization techniques by Saka and Erdal (2009), Kaveh and Talatahari (2010) and Dede (2013). The best result obtained for the second approach in this study is compared to the results of the other three studies in Table 4 to clarify the efficiency of SACO.

Table 4. Comparison of the results with the values in the literature

Algorithm	Design variables				Weight (kg)
	Group1	Group2	Group3	Group4	
SACO (This study, grouping b)	W200x15	W1000x222	W410x46.1	W460x52	7198
TLBO (Dede, 2013)	W760x147	W840x176	W150x13.5	W150x13.5	7131
CSS (Kaveh and Talatahari, 2010)	W150x13.5	W1000x222	W410x46.1	W460x52	7168
HSBO (Saka and Erdal, 2009)	W200x15	W1000x222	W410x46.1	W460x52	7198

The fittest solution of second grouping approach in this study are obtained as 7,198 kg. This solution is the same with those of Saka and Erdal (2009). But, the optimum solutions reached by Dede (2013) and Kaveh and Talatahari (2010) are better than the solution in this study for second grouping approach.

5. CONCLUSIONS

In this study, a simplified ant colony algorithm is used for size optimization of grillage structures to LRFD-AISC Manual of Steel Construction (1999). The purpose of the study is to clarify how the number of ants in colony effects the optimization process. For this purpose a grillage structure with different design variable grouping is optimized using various number of ants. The following conclusions can be drawn out at the end of the study.

The best result is obtained using the third grouping approach as expected; and this results is 14% lighter than the result of the first grouping approach. The better results are generally reached in the case of using more ants for all three grouping approaches.

The best result obtained in this study is either the same or very close to the results of the studies in the literature. There is a relationship between the number of ants required and the number of design variables. More ants must be used to achieve the optimum solution in the case of using more design variables. But, this relationship cannot be defined with a regular function. Additionally, although use of more ants reduces the number of iterations, the number of analyzes and the time required for the optimization process actually increases, depending on the number of ants. Therefore, it is possible to mention the optimum number of ants depending on the number of design variables and a preliminary analyze is required to determine this number.

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