European Journal of
Engineering and Applied Sciences

ISSN: 2651-3412 **Journal homepage:** dergipark.gov.tr/EJEAS

Published by Çorlu Faculty of Engineering, Namık Kemal University

Original Research Article

AC Power Formula for Unsaturated TiO_2 Memristors with Linear Dopant Drift, Small Signal AC Power Formula for All Memristors, and Some Applications for These Formulas

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Received: 13.07.2018 *Accepted: 22.11.2018*

Abstract: In 1971, Chua has claimed that there should have been one more fundamental circuit element called memristor. Memristor is nonlinear charge-dependent resistor. It dissipates power. No ideal memristor has been found yet. In 2008, a TiO_2 thin-film memristive system which behaves as a memristor for some part of its operation has been found by a HP research team. The new circuit component can allow new types of analog and digital applications which are not possible with other fundamental circuit elements. This has resulted an emerging interest in memristor and memristive systems. In this paper, the average power formula of TiO_2 memristor with linear dopant speed under AC excitation is derived. It is shown that a similar formula is applicable to all ideal memristors excited with a small signal AC source. The formulas derived here or similar formulas can be used to size the programmable memristor and memory circuits having AC waveforms.

Keywords: *Memristor, Memristor power, Memory power usage, Steady-state analysis of memristor*

Doymamış Lineer Sürüklenme Hızlı TiO_2 Memristörler İçin AC Güç Formülü, Tüm Memristörler İçin Küçük Sinyal AC Güç Formülü, ve Bu Formüllerin Bazı Uygulamaları

Özet: 1971 yılında, Dr. Chua, memristör adı verilen, bir tane daha temel devre elemanı olması gerektiğini iddia etti.Memristör yüke bağımlı nonlinear bir dirençtir. Memristör güç tüketir. Henüz ideal bir memristör bulunmamıştır. 2008'de, bir HP araştırma takımı tarafından çalışma bölgesininin bir kısmında memristörmüş gibi davranan TiO_2 ince-film bir memristif system bulunmuştur. Bu yeni devre elemanı diğer temel devre elemanları ile yapımı mümkün olmayan yeni tip analog ve sayısal uygulamalara izin verebilir. Bu ihtimal memristör ve memristif sistemler üzerine bir merak uyanmasını sağlamıştır. Bu makalede, AC gerilim altında, lineer sürüklenme hızlı TiO_2 memristörün ortalama güç formülü türetilmiştir. Benzeri formülün küçük sinyal AC kaynağından beslenen tüm ideal memristörlere uygulanabilirliği gösterilmiştir. Burada türetilen formüller ya da benzeri formüller AC gerilime sahip programlanabilir memristör ve hafıza devrelerinin boyutlandırılmasında kullanılabilir.

Anahtar Kelimeler: *Memristör, Memristör gücü, Hafiza güç tüketimi, Memristörün kararlı hal analizi*

Geliş: 13.07.2018 *Kabul: 22.11.2018*

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1. Introduction

The electrical circuits are thought to students as having three elements, resistor, inductor and capacitor, designated as R, L, and C. All the other components like diodes, transistors can be modeled with linear or nonlinear lumped components using those three components except independent or dependent power sources. The energy storing circuit elements are inductor and capacitor. Only Resistor from these three consumes power. One more circuit element called memristor, which is short for memory resistor, has been theoretically predicted by L. O. Chua in 1971 [1]. Memristor is a dissipative circuit element as resistor. Its value depends on charge which goes through it. Chua with Kang has also done some more research regarding the properties of memristors and memristive systems and predicted that memristor shows a zero-crossing pinched hysteresis curve when excited with AC current since the charge which goes through varies its value. [2]. For a long time, memristor is regarded as a mathematical work or a mathematical curiosity since no memristor was found. In 2008, a HP research team has declared that they have found a memristor in nanoscale made of TiO_2 sandwiched between Pt contacts [3]. They have also given a formula for memristance which is the memristor resistance for the first time in literature and were able to explain how it operates and the ionic mechanism behind it. If TiO_2 is fully doped, its memristor memristance takes its minimum value, the on-state resistance of the memristor. If the current has positive polarity and the memristor memristance is minimum, its memristor memristance stays the same that means memristor is under saturation. If TiO_2 is not doped at all, its memristor memristance takes its maximum value, the off-state resistance of the memristor. If the current has negative polarity and the memristor memristance is maximum, its memristor memristance stays the same that means memristor is under saturation. The memristor memristance takes values between the on-state and off-state resistances. They have also given the experimental hysteresis loop of the TiO_2 memristor [3]. A review paper on memristor can be found in [4].

New kind of memories can also be done using memristors [4] and most of the research on memristors is on memristor memories [6-10]. However, researches on analog applications of memristor have also started appearing on literature [11-14]. Adjustable or programmable gain applications of memristors are inspected in [11-14]. Programmable oscillators and programmable Schmitt-trigger circuits, and programmable threshold comparators using memristors are also considered in [12]. Usually, in the programmable circuit applications, memristor has AC voltage and current waveforms [11-14]. Also reading of memristor/memristive memories are done using AC waveforms not to change its stored charge [6-10]. Since it is a new circuit element, memristor power consumption should also be investigated. In 1971, Chua has shown memristor is a dissipative circuit element as resistor and also has proven that a memristor has no reactive power [1]. In literature, different types of memories are compared for their power consumption [7, 8, 10, and 16]. However, there are only a few papers regarding memristor power consumption [7, 8, 10, and 17]. In [17], TiO_2 memristor power consumption under AC excitation for linear and nonlinear drift model is investigated. However, numerical methods are used for TiO_2 memristor power calculation. The programmable circuits and memories with memristors should also be investigated for their power usage or energy consumption. This paper aims to fill the need. In this paper, to the best of our knowledge, the average power formula of unsaturated TiO_2 memristor with a linear dopant drift under AC excitation is derived for the first time in the literature. Power formulas for charge and flux dependent TiO_2 memristors are calculated. Also, power consumption of generic memristors under small signal AC excitation is derived. A way to calculate average memristance of a hysteresis curve is calculated. Examples are given for where to use the formulas. The formulas' applicability is shown.

TiO_2 memristor linear drift model is so often used in literature as it can be seen in the review paper and the references therein [4]. Linear dopant drift model of TiO_2 memristor given in [3] is also used in this paper. It is linearly dependent on charge if not saturated.

The paper is arranged as follows. In the second section, Charge and flux dependent TiO_2 memristor models with linear drift are given. In the third section, power loss of charge dependent TiO_2 memristor with Linear Dopant Drift s is derived. In the fourth section, power loss of flux dependent TiO_2 memristor with Linear Dopant Drift is derived. In the fifth section, power formulas of charge and flux dependent memristors fed by small AC signals are derived. In the sixth section, it is discussed on the power loss of Small Signal Hysteresis Loops. In the seventh section, a method for the calculation of average memristance of a memristor's hysteresis loop is done using the power formulas derived previously. In the eighth section, calculation of memristor loss of a programmable gain amplifier is done. In the ninth section, the calculation of reading loss of a memristor memory is done. The paper is concluded with the last section.

2. TiO_2 Memristor Model with Linear Dopant Drift

In this section, charge and flux dependent TiO_2 Memristor Models with Linear Dopant Drift are briefly explained. More information on models can be found in [3, 4].

2.1 Charge-dependent Memristance of TiO_2 Memristor

Memristance is designated as M(q) showing its charge dependency and it is equal to

$$\mathrm{M(q)} = \frac{d\lambda}{dq} = \frac{v(t)}{i(t)} \tag{1}$$

where

$\lambda(t)$ is the memristor flux, which is the integration of its voltage with respect to time.

$q(t)$ is the instantaneous memristor charge, which is the integration of its current with respect to time.

$v(t)$ is the memristor voltage.

$i(t)$ is the memristor current.

Memristance formula of TiO_2 memristor with linear dopant drift is given in [3];

$$\mathrm{M(q)} = R_{OFF}\left(1 - \frac{\mu_V R_{ON}}{D^2} q(t)\right) \quad (2)$$

where

R_{OFF} is the resistance of undoped TiO_2.

R_{ON} is the resistance of fully doped TiO_2.

D is the total length of the memristor.

μ_V is the mobility of oxygen atoms in the memristor.

The William's memristor memristance can also be written as

$$\mathrm{M(q)} = \mathrm{M_O} - Kq \quad (3)$$

where

$\mathrm{M(q)}$ is the memristor memristance.

$\mathrm{M_0}$ is the maximum memristance or the zero charge memristance and $\mathrm{M_0} = R_{OFF}$, which is the off-state resistance.

K is the memristor's charge coefficient.

If the memristor memristance takes its minimum value,

$$\mathrm{M_{sat}} = \mathrm{M(q_{sat})} = \mathrm{M_O} - Kq_{sat} \quad (4)$$

where $\mathrm{q_{sat}}$ is the saturation memristance charge and $\mathrm{M}_{sat} = R_{ON}$, which is the on-state resistance.

The following is always true for the HP memristor:

$$\mathrm{M_0} \geq \mathrm{M(q)} \geq \mathrm{M_{sat}} \quad (5)$$

2.2 Flux-dependent Memristance and Memductance of TiO_2 Memristor

TiO_2 Memristor Flux can be expressed as

$$\lambda = \int_0^q \mathrm{M(q)}dq = \mathrm{M_O}q - Kq^2/2 \quad (6)$$

This can be written as a quadratic equation;

$$Kq^2/2 - \mathrm{M_O}q + \lambda = 0 \quad (7)$$

The charge $q(t)$ is found as

$$q = \frac{-\mathrm{M_O} \pm \sqrt{\mathrm{M_0^2} - 2K\lambda}}{K} \quad (8)$$

Its time derivative is equal to the TiO_2 memristor current;

$$i(t) = \frac{dq(t)}{dt} = \frac{V(t)}{\sqrt{\mathrm{M_0^2} - 2K\lambda}} \quad (9)$$

We have only retained the sign "+" in "+-" so that the solution is consistent with the reality.

Although it is traditionally assumed that memductance is a function of memristor flux and memristance is a function of memristor charge, the memristor memristance can also be expressed as a function of memristor flux;

$$M(\lambda) = \sqrt{\mathrm{M_0^2} - 2K\lambda} \quad (10)$$

The TiO_2 memductance is then equal to

$$\psi(\lambda) = \frac{1}{M(\lambda)} = \frac{1}{\sqrt{\mathrm{M_0^2} - 2K\lambda}} \quad (11)$$

The memductance formula is used in the fourth section.

3. Power Loss of Charge Dependent TiO_2 Memristor with Linear Dopant Drift

Let's assume the memristor is fed by an AC current and it is in steady state as shown in Figure 1. Therefore, current, voltage, and charge of memristor are all periodic. Let's assume that memristor charge as a function of time is equal to

$$q(t) = \overline{Q} - \sum_{k=1}^{\infty} q_k \cos(k\omega_e t + \alpha_k) \quad (12)$$

Where

$\overline{Q}$ is the average charge of memristor in steady-state.

q_k is the magnitude of the k^{th} harmonic of the charge.

α_k is the phase angle of k^{th} harmonic of the charge.

ω_e is the angular speed.

T_e is the period of the AC current.

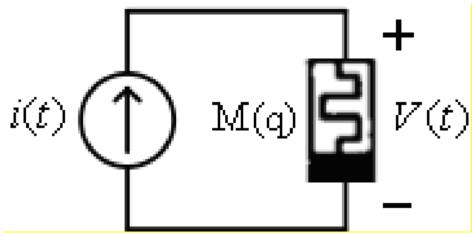


Figure 1. Memristor driven by a current source.

The memristor current is

$$i(t) = \frac{dq}{dt} = \sum_{k=1}^{\infty} q_k \sin(k\omega_e t + \alpha_k) \quad (13)$$

The memristor power loss in steady-state is

$$\overline{\text{p}}=\frac{\int_{t_0}^{t_0+T_e} \text{M(q(t))i}^2\text{(t)}dt}{T_e} \quad (14)$$

$$\overline{\text{p}}=\int_{t_0}^{t_0+T_e} \frac{\left(M_o - K\overline{Q} + K\sum_{k=1}^{\infty} q_k \cos(k\omega_e t + \alpha_k)\right)\left(\sum_{k=1}^{\infty} q_k \sin(k\omega_e t + \alpha_k)\right)^2}{T_e} dt \quad (15)$$

$$\overline{\text{p}}=\int_{t_0}^{t_0+T_e} \frac{\left(M_o - K\overline{Q}\right)\left(\sum_{k=1}^{\infty} \text{k}\omega_\text{e}\text{q}_\text{k}\text{sin(k}\omega_\text{e}\text{t+}\alpha_k)\right)^2}{T_e} dt \quad (16)$$

$$\overline{\text{p}}=\left(M_o - K\overline{Q}\right) I_{RMS}^2 \quad (17)$$

since

$$\int_{t_0}^{t_0+T_e} \sum_{k=1}^{\infty} \text{q}_\text{k}\text{cos(k}\omega_\text{e}\text{t+}\alpha_k)\left(\sum_{k=1}^{\infty} \text{k}\omega_\text{e}\text{q}_\text{k}\text{sin(k}\omega_\text{e}\text{t+}\alpha_k)\right)^2 dt \quad (18)$$

I_{RMS} is the rms value of the memristor current and equal to

$$I_{RMS} = \sqrt{\text{f}_e \int_{t_0}^{t_0+T_e} i(t)^2 \, dt} \quad (19)$$

These followings can be inferred from (17).

1. The average power of TiO_2 memristor does not depend on frequency of AC signal.

2. The average power of William's memristor does depend on its average charge and on its rms current.

3. $\text{M}(\overline{\text{Q}}) = \text{M}_\text{o}\text{-K}\overline{\text{Q}}$, is the value of the equivalent resistor which dissipates the same amount of power for the same rms value of current. The equivalent resistor, which dissipates the same power, does depend on the average memristor charge.

4. Power Loss of Flux Dependent TiO_2 Memristor with Linear Dopant Drift

Now, the memristor power formula for a known memristor current is derived and another formula for a given memristor flux (or voltage) can be found. Let's assume the memristor is fed by an AC voltage source and it is in steady state as shown in Figure 2. Therefore, current, voltage, charge and flux of the memristor are all periodic. Let's assume that memristor flux as a function of time is equal to

$$\lambda(t) = \overline{\lambda} - \sum_{k=1}^{\infty} \lambda_k \cos(k\omega_e t + \varphi_k) \quad (20)$$

Where

$\overline{\lambda}$ is the average flux of memristor in steady-state.

λ_k is the magnitude of the k^{th} flux ripple harmonic.

ϕ_k is the phase angle of k^{th} charge ripple harmonic.

ω_e is the angular speed.

T_e is the period of the AC voltage.

The memristor voltage is

$$v(t) = \sum_{k=1}^{\infty} k\omega_e \lambda_k \sin(k\omega_e t + \varphi_k) \quad (21)$$

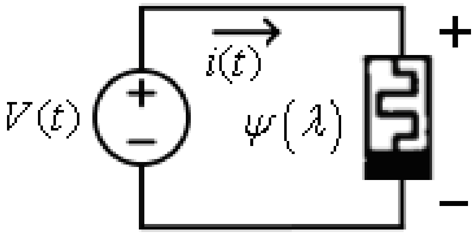


Figure 2: Memristor driven by a voltage source

Now, by using Equations (10), (20) and (21), the memristor power loss in steady-state is

$$\overline{\text{p}}=\text{f}_e \int_{t_0}^{t_0+T_e} \frac{v^2\text{(t)}}{\text{M}(\lambda\text{(t)})} dt = \int_{t_0}^{t_0+T_e} \frac{v^2\text{(t)}dt}{T_e\sqrt{M_0^2 - 2K\lambda}} \quad (22)$$

The memristance can be approximated as

$$\frac{1}{\sqrt{M_0^2 - 2K\lambda}} \cong \frac{1}{M_0} - \frac{K\overline{\lambda} + K\lambda}{M_0^2} \quad (23)$$

The memristor power is

$$\overline{\text{p}}=\text{f}_e \int_{t_0}^{t_0+T_e} \left(\frac{1}{M_0} - \frac{K\overline{\lambda}+K\lambda}{M_0^2} + \frac{K}{M_0^2}\sum_{k=1}^{\infty} \lambda_\text{k}\text{cos(k}\omega_\text{e}\text{t+}\phi_k)\right) .\left(\sum_{k=1}^{\infty} \text{k}\omega_\text{e}\lambda_\text{k}\text{sin(k}\omega_\text{e}\text{t+}\phi_k)\right)^2 dt \quad (24)$$

$$\overline{\text{p}}=\text{f}_e \int_{t_0}^{t_0+T_e} \left(\frac{1}{\sqrt{M_0^2 - 2K\lambda_0}} + K\left(\lambda - \overline{\lambda}\right)\right). \left(\sum_{k=1}^{\infty} \text{k}\omega_\text{e}\lambda_\text{k}\text{sin(k}\omega_\text{e}\text{t+}\phi_k)\right)^2 dt \quad (25)$$

$$\overline{\text{p}}=\frac{V_{RMS}^2}{\sqrt{M_0^2 - 2K\overline{\lambda}}} \quad (26)$$

since

$$\int_{t_0}^{t_0+T_e} \sum_{k=1}^{\infty} \lambda_\text{k}\text{os(k}\omega_\text{e}\text{t+}\phi_k)\left(\sum_{k=1}^{\infty} \text{k}\omega_\text{e}\lambda_\text{k}\text{sin(k}\omega_\text{e}\text{t+}\phi_k)\right)^2 dt = 0. \quad (27)$$

V_{RMS} is the rms value of the memristor voltage and equal to

$$V_{RMS} = \sqrt{f_e \int_{t_0}^{t_0+T_e} v(t)^2 dt} \,. \qquad \textbf{(28)}$$

These followings can be inferred from (28).

1.The average power of TiO_2 memristor does not depend on frequency of AC signal.

2.The average power of HP memristor does depend on its average flux and its rms voltage.

3. $M\left(\overline{\lambda}\right) = \sqrt{M_0^2 - 2K\overline{\lambda}}$, is the value of the equivalent resistor which dissipates the same amount of power for the same rms value of voltage. The equivalent resistor, which dissipates the same power, does depend on the average memristor flux.

5. Application of These Power Formulas to Memristors Fed by Small Signals

In [15], Taylor series is used to express memristance function of a memristor as a function of charge and used to explain memristor's hysteresis loops for small signals. Therefore, memristance of any unsaturated memristor can be written as

$$M(q) = M_0 + \sum_{j=1}^{\infty} \frac{\partial M^j(q_0)}{\partial q^j} \frac{(q-q_0)^j}{j!} \qquad (29)$$

And memductance of any unsaturated memristor is

$$\psi(\lambda) = \psi_0 + \sum_{j=1}^{\infty} \frac{\partial \psi^j(\lambda_0)}{\partial \lambda^j} \frac{(\lambda-\lambda_0)^j}{j!} \qquad (30)$$

The series for small signal can be approximated as

$$M(q) \cong M_0 + \frac{\partial M(q_0)}{\partial q}(q-q_0) \qquad (31)$$

and

$$\psi(\lambda) \cong \psi_0 + \frac{\partial \psi(\lambda_0)}{\partial \lambda}(\lambda-\lambda_0) \qquad (32)$$

Since these formulas, Equations (31) and (32) are similar to (3), their power dissipation formulas are found as similar to (17) and (26) for small AC signals;

$$p_{LOSS} = M(\overline{Q}) I_{RMS}^2 \qquad (33)$$

and

$$p_{LOSS} = \psi(\overline{\lambda}) V_{RMS}^2 \qquad (34)$$

Again M_0 and ψ_0 are average memristance and average memductance in a period. Therefore, the power formulas (33) and (34) are applicable to any memristor with no other dependency than charge of flux excited with small signal AC sources.

6. On the Power Loss of Small Signal Hysteresis Loops of Memristors

Memristor has a hysteresis loop when excited with AC current since the charge which goes through varies its value [2]. Chua's hypothetical hysteresis loop given in [2] is shown in Figure 3. The hysteresis curve turns into a straight line with the increasing frequency as shown in Figure 3. All three hysteresis curves have the same average charge. There is a very interesting property of small signal hysteresis loops inferred from the power formulas found; all the hysteresis loops given in Figure 3 have the same average power for the same average charge (or the same average flux) for small signal AC excitation.

7. Calculation of Average Memristance of Small Signal Hysteresis Loops of Memristors

Using these formulas, the average memristance of TiO_2 memristor excited by an AC current source or any memristor excited by a small signal AC voltage source can be found.

These two formulas give the same power dissipation value for the same hysteresis loop;

$$\overline{p} = \frac{V_{RMS}^2}{\sqrt{M_0^2 - 2K\overline{\lambda}}} \,, \qquad (35)$$

$$\overline{p} = \left(M_o - K\overline{Q}\right) I_{RMS}^2 \,, \qquad (36)$$

and

$$\overline{p} = \frac{V_{RMS}^2}{\sqrt{M_0^2 - 2K\overline{\lambda}}} = \left(M_o - K\overline{Q}\right) I_{RMS}^2 \qquad (37)$$

Remembering

$$\mathrm{M}(\overline{\mathrm{Q}}) = \mathrm{M}(\overline{\lambda}) = \sqrt{M_0^2 - 2K\lambda} = M_0 - K\overline{Q} \qquad (38)$$

The average memristance of a period is found as

$$\mathrm{M}(\overline{\mathrm{Q}}) = \mathrm{M}(\overline{\lambda}) = \frac{V_{RMS}}{I_{RMS}} \qquad (39)$$

Therefore, the average memristance of a memristor under AC excitation for a small-signal applied (the average memristance of the hysteresis loops in Figure 3) can be calculated from the rms values of both its current and voltage. If the memristor parameters are known, its average charge or average flux of the concerned hysteresis loop can be calculated easily as

$$\overline{Q} = \frac{M_o - \mathrm{M}(\overline{\mathrm{Q}})}{K} \qquad (40)$$

and

$$\overline{\lambda} = \frac{M_0^2 - \left(\mathrm{M}(\overline{\lambda})\right)^2}{2K} \qquad (41)$$

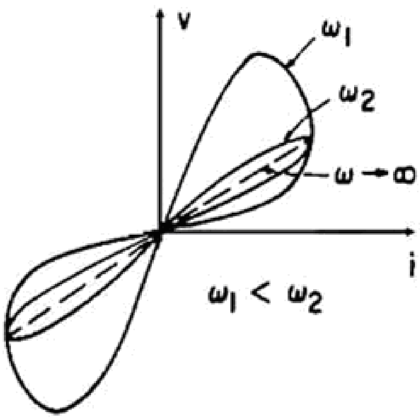
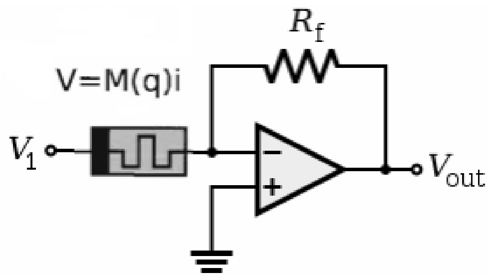


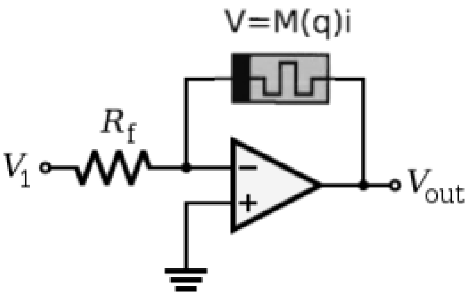
Figure 3. Leon Chua’s Original Graph of a Hypothetical Memristor’s Hysteresis Curve [2].

8. Calculation of Memristor Loss of a Programmable Gain Amplifier

Now, we will give an example to calculate power loss of a memristor used in a programmable amplifier. Inverting gain amplifier with a memristor is shown in Figure 4.a. Another inverting gain amplifier with a memristor is shown in Figure 4.b.



(a)



(b)

Figure 4. a) M-R Topology, b) R-M Topology.

The power loss of the memristor in Figure 4.a for small input voltage $\boldsymbol{V_1}$ is equal to

$$\overline{\mathrm{p}} = \frac{V_{RMS}^2}{\sqrt{M_0^2 - 2K\overline{\lambda}}} \qquad \textbf{(42)}$$

or

$$\overline{\mathrm{p}} = \frac{V_{RMS}^2}{\sqrt{M_0^2 - 2K\overline{\lambda}}} = \frac{V_{RMS}^2}{M_o - K\overline{Q}} \qquad (43)$$

The input current of the R-M Programmable Gain Amplifier is equal to the memristor current is in Figure 4.b and the input current is equal to

$$i_1 = V_1 / R \qquad (44)$$

and the rms value of memristor current is

$$I_{RMS} = \sqrt{\mathrm{f}_e \int_{t_0}^{t_0+T_e} i^2 dt} \qquad (45)$$

$$I_{RMS} = V_{1rms} / R_1 \qquad (46)$$

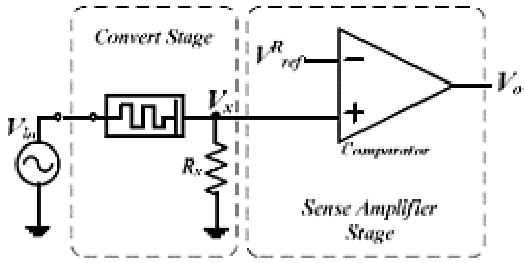
The power loss of the memristor in Figure 4.b for input voltage $\boldsymbol{V_1}$ is equal to

$$\overline{\mathrm{p}} = \left(M_o - K\overline{Q}\right) I_{RMS}^2 \qquad (47)$$

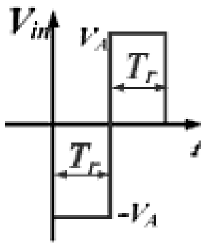
Therefore, the formulas we have found can be used for power loss analysis of memristors in programmable memristor circuits, too.

9. Calculation of Reading Loss of a Memory

The power loss formulas obtained previously is usable for calculation of memory power or energy usage. The memory circuit given in [8] is shown in Figure 5. Possible Usage Areas for the Power Formulas are for programmable devices, programmable resistive circuits, and memory Circuits.



(a)



(b)

Figure 5. a) The operation stages of memory circuit in [8]. b) Read pattern.

For the waveform of the reading voltage given in Figure 3, the rms voltage of reading waveform is

$$V_{RMS} = \sqrt{f_e \int_{t_0}^{t_0+T_e} v(t)^2\, dt} \qquad (48)$$

$$V_{RMS} = \sqrt{2.T_r.f_e}.V_{dc} \qquad (49)$$

Remembering, the resistor and memristor are connected in series, therefore, the total power loss of the memristor reading is

$$p(t) = \frac{V^2(t)}{M_0 + R - Kq} \qquad (50)$$

For a small signal (For low power loss, rms value of the signal should be small, too), the power loss of reading of memristor memory is found submitting ***R+Mo*** in ***Mo*** in (44);

$$\overline{p} = \frac{V_{RMS}^2}{M_0 + R - K\overline{q}} \qquad (51)$$

or

$$\overline{p} = \frac{V_{RMS}^2}{R + M(\overline{q})} \qquad (52)$$

where

$$M(\overline{q}) = M_O - K\overline{q} \qquad (53)$$

If q_1 is the charge value of the memristor memory set for Logic 1, the average memristance at logic 1 can be taken as

$$M(q_1) = M_O - 0.75Kq_{sat} \qquad (54)$$

If q_2 is the charge value of the memristor memory set for Logic 0, the memristance at logic 0 is

$$M(q_2) = M_O - 0.25Kq_{sat} \qquad (55)$$

Power loss of Memristor-resistor series circuit for reading of Logic I;

$$P_{LOSS\_LOGIC1} = \frac{V_{RMS}^2}{R + M(q_1)} \qquad (56)$$

The energy loss for a reading logic 1 is found as

$$E_{LOSS\_LOGIC1} = P_{LOSS\_LOGIC1} T_e \qquad (57)$$

Power loss of Memristor-resistor series circuit for reading of Logic 0;

$$P_{LOSS\_LOGIC0} = \frac{V_{RMS}^2}{R + M(q_2)} \qquad (58)$$

The energy loss for a reading logic 0 is found as

$$E_{LOSS\_LOGIC0} = P_{LOSS\_LOGIC0} T_e \qquad (59)$$

Reviewing (57) and (59), one can see that Reading loss of Logic 1 is greater than that of Logic 2. The power formulas may be usable at other type of memristor memories, too.

10. Conclusion

Power loss formula for William's Memristor is derived. It is interpreted. Memristor power is found to be independent of frequency and dependent on memristor parameters, average memristor charge and rms memristor current. The formula is not applicable to saturated memristor under AC excitation. Being able to use the formula, the memristor charge should be kept between zero and the saturation charge.

In future, when more types of memristors are found or when memristors are understood more, there may have to be more parameters to consider such as current and temperature dependency.

Also, the power formulas are used to derive a formula for calculation of average memristance of a memristor under small signal AC excitation.

Time is going to show whether a paradigm shift will occur or not to accept Memristor as the fourth circuit element. Yet, it is important to do analyze it and its properties as much as possible now with the mathematical tools, we have, so that we can make the best use of memristors for or when they will be commercially available. This work is intended to be a contribution to the search of an alternative way for information storage and expected to be useful in research towards nanoscale memories. It is hoped that the formulas can be used for pre-sizing of memristor memories or memristor programmable devices.

References

[1] Chua, L. (1971). Memristor-the missing circuit element. IEEE Transactions on circuit theory, 18(5), 507-519.

[2] Chua, L. O., & Kang, S. M. (1976). Memristive devices and systems. Proceedings of the IEEE, 64(2), 209-223.

[3] Strukov, D. B., Snider, G. S., Stewart, D. R., & Williams, R. S. (2008). The missing memristor found. nature, 453(7191), 80.

[4] Kavehei, O., Iqbal, A., Kim, Y. S., Eshraghian, K., Al-Sarawi, S. F., & Abbott, D. (2010). The fourth element: characteristics, modelling and electromagnetic theory of the memristor. In Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences(Vol. 466, No. 2120, pp. 2175-2202). The Royal Society.

[5] Manem, H., Rose, G. S., He, X., & Wang, W. (2010, May). Design considerations for variation tolerant multilevel CMOS/Nano memristor memory. In Proceedings of the 20th Great lakes symposium on VLSI (pp. 287-292). ACM.

[6] Huang, G. M., Ho, Y., & Li, P. (2010). Memristor system properties and its design applications to circuits such as nonvolatile memristor memories. In International Conference on Communications, Circuits and Systems (ICCCAS), 2010 (pp. 805-810). IEEE.

[7] Ho, Y., Huang, G. M., & Li, P. (2011). Dynamical properties and design analysis for nonvolatile memristor memories. IEEE Transactions on Circuits and Systems I: Regular Papers, 58(4), 724-736.

[8] Niu, D., Chen, Y., & Xie, Y. (2010, August). Low-power dual-element memristor based memory design. In Proceedings of the 16th ACM/IEEE international symposium on Low power electronics and design (pp. 25-30). ACM.

[9] Vontobel, P. O., Robinett, W., Kuekes, P. J., Stewart, D. R., Straznicky, J., & Williams, R. S. (2009). Writing to and reading from a nano-scale crossbar memory based on memristors. Nanotechnology, 20(42), 425204.

[10] Jo, K. H., Jung, C. M., Min, K. S., & Kang, S. M. (2010). Self-adaptive write circuit for low-power and variation-tolerant memristors. IEEE Transactions on Nanotechnology, 9(6), 675-678.

[11] Wey, T., & Jemison, W. (2012). An automatic gain control circuit with TiO2 memristor variable gain amplifier. Analog Integrated Circuits and Signal Processing, 73(3), 663-672.

[12] Pershin, Y. V., & Di Ventra, M. (2010). Practical approach to programmable analog circuits with memristors. IEEE Transactions on Circuits and Systems I: Regular Papers, 57(8), 1857-1864.

[13] Shin, S., Kim, K., & Kang, S. M. (2009, July). Memristor-based fine resolution programmable resistance and its applications. In International Conference on Communications, Circuits and Systems, 2009. (pp. 948-951). IEEE.

[14] Shin, S., Kim, K., & Kang, S. M. (2011). Memristor applications for programmable analog ICs. IEEE Transactions on Nanotechnology, 10(2), 266-274.

[15] MUTLU R. (2010). Taylor Serisi ve Kutupsal Fonksiyonlar Kullanarak Memristorün (Hafızalı Direncin) Histeresis Eğrisinin Açıklanması. 3. Ileri Muhendislik Teknolojileri Sempozyumu, 29-30 Mayis 2010. Cankaya Universitesi, Ankara, Turkey.

[16] Guo, X., Ipek, E., & Soyata, T. (2010, June). Resistive computation: avoiding the power wall with low-leakage, STT-MRAM based computing. In ACM SIGARCH Computer Architecture News (Vol. 38, No. 3, pp. 371-382). ACM.

[17] Gazabare, S., Pieper, R. J., Wondmagegn, W., & Satyala, N. (2011, March). Observations on model based predictions for memristor power dissipation. In Southeastcon, 2011 Proceedings of IEEE (pp. 450-454). IEEE.