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Classification of Robinson-Trautman and Kundt geometries with Large D limit

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Abstract: Algebraic classification of higher dimensional, shear-free, twist-free, expanding (or non-expanding) spacetime is studied with the limit of $D \to \infty$. Similar to classification of any arbitrary dimension $D > 4$, this spacetime is Type I(b) or more special, according to our calculations. However, thanks to the method of taking the limit of dimension $D \to \infty$, the relevant Weyl scalars become much simpler. Without solving field equations, by determining obligatory conditions to the components of Weyl scalar vanish, the spacetime is classified Type I(a), Type II(a-b-c-d), Type III(a-b), Type N and Type O for primary Weyl aligned null direciton (WAND), and Type I_i, Type II_i, Type III_i and Type D(a-b-c-d) for secondary WAND.

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Contents

1 Introduction

Higher dimensional spacetimes are commonly studied to unify string theory and general relativity and to analyze the quantum field theories with CFT/AdS correspondence and to fully understand the efficiency of the theory of general relativity. Nearly during past decade, Emparan and et al... examine the solutions of higher dimensional spacetimes in the limit of dimension $D \to \infty$ [1–10]. The obtained results make it easier to gain a new perspective on theory of general relativity and especially analytical solutions, but it is also tested in order to be used in some other areas, i.e. quantum entanglement and holographics [11].

Robinson-Trautman (RT) [12, 13] and Kundt [14, 15] spacetimes are special spacetimes which include many different solutions in 4-dimensions such as, pp-waves, plane wave spacetimes, the Schwarzschild and Reissner-Nordstrom black holes, the C-metric, the Vaidya solution, photon rockets and their non-rotating generalisations. RT spacetime is described by the existence of an expanding, shearfree and twistfree congruence of null geodesics, while, Kundt geometry is defined as a non-expanding case. Although, higher-dimensional solutions are not as rich as 4-dimensional solutions because several important solutions can not be generalized to higher dimensions. However, extension of RT spacetime to higher dimensions were studied for any cosmological constant or aligned pure radiation [16], aligned electromagnetic fields [17], and general p-form fields [18]. Additionally, the Kundt spacetime generalization to higher dimensions is obtained in [19].

Classification of spacetimes, which enables to divide the gravitational fields into distinct types in an invariant way, was first studied by Petrov [20]. There are several ways to obtain the classification of spacetimes such as, null vectors, and 2-spinors or scalar invariants.

Since the Weyl scalar and cosmological constant include all data about curvature of a spacetime, the classification becomes completely understandable by studying Weyl scalars. Classification provided a better understanding of several aspects of General Relativity in 4-dimensions and it was extended to any $D > 4$ dimensions [21] and to RT [22] and Kundt [23] geometries as well. However, the algebraic classification of the RT and Kundt spacetimes with the limit of $D \to \infty$ have not been studied before.

Therefore, the main purpose of the paper is to obtain algebraic classifications of RT and Kundt spacetimes in the limit of $D \to \infty$ and compare the results with previous calculations of any dimension $D > 4$.

The paper is organized as; section 2 includes a brief summary of shearless and twistless, expanding or non-expanding spacetimes. Primary and secondary Weyl-aligned null direction (WAND) are reviewed and the classification for components of boost weight are demonstrated. Components of Weyl scalars are obtained with the limit of $D \to \infty$ and the different types of spacetimes are discussed. In section 3 and 4 algebraic classification of the non-expanding Kundt spacetime and expanding RT spacetime are stated and several special cases are investigated for the limit of $D \to \infty$, respectively. Christoffel symbols, Riemann and Ricci tensor and Ricci scalar are calculated for the general expanding, shearless, twistfree spacetime for any dimension $D > 4$, while the components of Weyl scalar are computed for the limit $D \to \infty$ in appendix A.

2 Non-twisting, shear-free geometry

In general, D-dimensional, shear-free and twist-free metric can be written in the following form;

$$ds^2 = g_{pq}\,(u,r,x)\,dx^p dx^q + 2g_{up}\,(u,r,x)\,dudx^p - 2dudr + g_{uu}\,(u,r,x)\,du^2 \tag{2.1}$$

where latin indices $p, q, \ldots$ count to 2 to $(D-2)$ and x is shorthand of these $D-2$ spatial coordinates on the traverse space. We can write some of the relations between contravariant and covariant metric components as;

$$g^{ur} = -1, \qquad g^{rp} = g^{pq} g_{uq}, \qquad g^{rr} = -g_{uu} + g^{pq} g_{up} g_{uq}, \qquad g_{up} = g^{rq} g_{pq}.$$

Non-twisting spacetimes require a null hypersurface as u =constant which is similar to existence of a null vector field $\mathbf{k}$ that is tangent to u=constant surface in everywhere (it is more common to choose $\mathbf{k}$ at r direction and it becomes $\mathbf{k} = \partial_r$.). This null vector covariant derivative for the metric (2.1) is obtained $k_{i;j} = \Gamma^u{}_{ij} = \frac{1}{2} g_{ij,r}$ which satisfies $k_{r;j} = k_{i;r} = 0$. Another orthonormal basis in the $D-2$ dimensional spatial coordinates can be defined as m_i^p which is useful to identify optical matrix; $\rho_{ij} = k_{p;q} m_i^p m_j^q$. For shearless and twist-free spacetimes reduce to; $\rho_{ij} = \Theta\delta_{ij}$. The expansion scalar is obtained $2\Theta g_{pq} = g_{pq,r}$ by the definition of $\delta_{ij} = g_{pq} m_i^p m_j^q$ [22]. The vanishing expansion, $\Theta = 0$ is known as Kundt class [14, 15, 24, 25] which indicates the condition that the spatial metric is independent of the affine parameter r (in metric (2.1) $g_{pq}(u,r,x) \to g_{pq}(u,x)$). Otherwise, expanding case, $\Theta \neq 0$ is named as Robinson-Trautman class [12, 13, 24, 25].

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	Boost Weight Components					
Types	+2	+1	0	-1	-2	primary WAND
G						No
I	0					a WAND
II	0	0				multiple WAND
III	0	0	0			multiple WAND
N	0	0	0	0		multiple WAND
O	0	0	0	0	0	multiple WAND

Table 1. Higher dimensional classification of a spacetime with obligatory vanishing components of boost weights.

We begin with identify natural null frames to obtain classification of the spacetime which is given metric (2.1) as;

$$\mathbf{k} = \partial_r, \qquad \ell = \frac{1}{2} g_{uu}\partial_r + \partial_u, \qquad \mathbf{m_i} = m_i^p \left(g_{up}\partial_r + \partial_p \right), \tag{2.2}$$

which satify the normalization conditions; $\mathbf{k}.\ell = -1$, $\mathbf{m_i}.\mathbf{m_j} = \delta_{ij}$. Boosts are defined by rescaling of these null basis as; $\mathbf{k} \to \lambda\mathbf{k}$, $\ell \to \lambda^{-1}\ell$ and $\mathbf{m}_i \to \mathbf{m}_i$ and boost weight which is used to determine the classification of the spacetimes in higher dimensions, are obtained $+1, -1, 0$, respectively. Additionally, if the boost weight $+2$ components of the Weyl tensor are zero, the null direction of $\mathbf{k}$ becomes a primary Weyl-aligned null direction (WAND) which is analogue of a PND in 4-dimensions. There might be multiple WAND if the $+1$ boost weight components of the Weyl scalar vanish and if the spacetime obeys multiple WAND conditions, it becomes algebraically special. One prepared the classification of the spacetime with obligatory vanishing components of boost weight [26] which is summarized in table 1. Further, for fixed $\mathbf{k}$, ℓ can be introduced a secondary WAND which can be obtained by vanishing as many as null frame scalars. According to secondary WAND the spacetime will be Type I_i, II_i, III_i and Type D.

Subsequently, as the dimension of the spacetime goes to infinity, the Weyl scalars of the metric (2.1) become (Christoffel symbols, Riemann and Ricci tensors, Ricci scalar and Weyl tensor are calculated at the appendix A),

$$\Psi_{0^{ij}} = C_{abcd}k^a m_i^b k^c m_j^d = m_i^p m_j^q C_{rprq} = 0, \tag{2.3}$$

$$\Psi_{1T^i} = C_{abcd}k^a \ell^b k^c m_i^d = m_i^p C_{rurp} = m_i^p \left[\left(-\frac{1}{2} g_{up,r} + \Theta g_{up} \right)_{,r} + \Theta_{,p} \right], \tag{2.4}$$

$$\Psi_{1^{ijk}} = C_{abcd}k^a m_i^b m_j^c m_k^d = m^p_{,} m_j^q m_k^m C_{prmq} = 0, \tag{2.5}$$

$$\begin{aligned} \Psi_{2S} = C_{abcd}k^a \ell^b \ell^c k^d = C_{ruur} = &\left(\frac{1}{2} g_{uu,r} - \Theta g_{uu} \right)_{,r} - \frac{1}{4} g^{pq} g_{up,r} g_{uq,r} - 2\Theta_{,u} \\ &+ \Theta g^{rp} g_{up,r} - \Theta^2 g^{rp} g_{up}, \end{aligned} \tag{2.6}$$

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$$
\begin{aligned}
\Psi_{2T^{ij}} &= C_{abcd}k^a m_i^b \ell^c m_j^d = m_i^p m_j^q \left(C_{rpuq} + g_{up}C_{rqur} + \frac{1}{2}g_{uu}C_{rprq} \right) \\
&= m_i^p m_j^q \bigg(\frac{1}{2}g_{up}g_{uq,rr} + \frac{1}{4}g_{up,r}g_{uq,r} + \frac{1}{2}g_{pn}g^{ms}g_{us,r}\ {}^s\Gamma^n{}_{mq} + \frac{1}{2}g_{pn}\left(g^{nm}g_{um,r}\right)_{,q} \\
&\quad + g_{up}g_{uq}\left(\Theta^2 - \Theta_{,r}\right) - 2g_{up}\Theta_{,q} - \Theta\left(2E_{qp} - g_{uq}g_{up,r}\right) \bigg),
\end{aligned} \tag{2.7}
$$

$$
\begin{aligned}
\Psi_{2^{ijkl}} &= C_{abcd}m_i^a m_j^b m_k^c m_l^d = m_i^p m_j^q m_k^n m_l^m \left(C_{pqmn} + g_{up}C_{rqmn} + g_{uq}C_{prmn} + g_{um}C_{pqrn} \right. \\
&\quad \left. + g_{un}C_{pqmr} \right) = m_i^p m_j^q m_k^n m_l^m C_{pqnm},
\end{aligned} \tag{2.8}
$$

$$
\begin{aligned}
\Psi_{2^{ij}} &= C_{abcd}k^a \ell^b m_i^c m_j^d = m_i^p m_j^q \left(C_{rupq} + g_{uq}C_{rurp} + g_{up}C_{ruqr} \right) = m_i^p m_j^q \bigg(g_{u[p,q]r} \\
&\quad - 4g_{u[p}\Theta_{,q]} + g_{u[p}g_{q]u,rr} + \Theta\left(2g_{u[q}g_{p]u,r} + E_{qm} - E_{pn}\right) \bigg),
\end{aligned} \tag{2.9}
$$

$$
\begin{aligned}
\Psi_{3T^i} &= C_{abcd}\ell^a k^b \ell^c m_i^d = m_i^p \left(\frac{1}{2}g_{uu}C_{urrp} + g_{up}C_{urur} + C_{urup} \right) \\
&= m_i^p \bigg(\frac{1}{4}g_{uu}g_{up,rr} - g_{u[u,p]r} + g_{up}\Theta_{,u} + \frac{1}{2}g_{uu}\Theta_{,p} - \frac{1}{2}g_{uu}g_{up}\Theta_{,r} + \frac{1}{2}g^{mn}g_{um,r}E_{np} \\
&\quad - g_{up}\left(\frac{1}{2}g_{uu,r} - \Theta g_{uu}\right)_{,r} - \frac{\Theta}{2}\left(g^{rr}g_{up,r} + g_{uu,p} + 2g^{rs}E_{sp}\right) \bigg),
\end{aligned} \tag{2.10}
$$

$$
\begin{aligned}
\Psi_{3^{ijk}} &= C_{abcd}\ell^a m_i^b m_j^c m_k^d = m_i^p m_j^q m_k^m \bigg(\frac{1}{2}g_{uu}\left(C_{rpqm} + g_{uq}C_{rprm} + g_{um}C_{rpqr}\right) \\
&\quad + g_{up}\left(C_{urqm} + g_{uq}C_{urrm} + g_{um}C_{urqr}\right) + g_{uq}C_{uprm} + g_{um}C_{upqr} + C_{upqm} \bigg) \\
&= m_i^p m_j^q m_k^m \bigg(-2g_{up}g_{u[q,m]r} - \Theta g_{up}\left(E_{mn} - E_{qs}\right) + g_{up}g_{u[q}g_{m]u,rr} + g_{up}g_{u[q}g_{m]u,r} \\
&\quad + g^{\ell s}g_{us,r}\ {}^s\Gamma^n{}_{\ell p}g_{u[q}g_{m]n} + \frac{1}{2}g_{up,r}g_{u[q}g_{m]u,r} + g_{u[q}g_{m]n}\left(g^{n\ell}g_{u\ell,r}\right)_{,p} - 4\Theta E_{p[m}g_{q]u} \\
&\quad - g_{pn}\left(g^{rn}g_{u[m,r}\right)_{,q]} - E_{p[m}g_{q]u,r} - 2g_{pn}\left(g^{ns}E_{s[q}\right)_{,m]} - g_{pn}g^{rs}\ {}^s\Gamma^n{}_{s[q}g_{m]u,r} \\
&\quad - 2g_{pn}g^{sk}E_{k[q}\ {}^s\Gamma^n{}_{m]s} - 2\Theta^2 g^{rr}g_{p[q}g_{m]u} \bigg),
\end{aligned} \tag{2.11}
$$

$$
\begin{aligned}
\Psi_{4^{ij}} &= C_{abcd}\ell^a m_i^b \ell^c m_j^d = m_i^p m_j^q \bigg(\frac{g_{uu}}{2}\left(C_{rpuq} + C_{uprq} + \frac{g_{uu}}{2}C_{rprq} + g_{uq}C_{rpur} + g_{up}C_{urrq} \right) \\
&\quad + g_{up}\left(C_{uruq} + g_{uq}C_{urur}\right) + g_{uq}C_{upur} + C_{upuq} \bigg) = m_i^p m_j^q \bigg(-g_{uq}g_{u[u,p]r} \\
&\quad + \frac{g_{uu}}{2}\left(g^{ms}g_{us,r}\ {}^s\Gamma^n{}_{m(p}g_{q)n} + \left(g^{nm}g_{um,r}\right)_{,(p}g_{q)n} - 4\Theta E_{(pq)} \right) \\
&\quad + g^{mn}g_{um,r}E_{n(p}g_{q)u} - \frac{g^{rm}}{2}g_{um,r}g_{u(p}g_{q)u,r} + g_{up}g_{uq}\left(\frac{1}{2}g^{pq}g_{up,r}g_{uq,r} + \Theta g_{uu,r}\right) \\
&\quad - g_{pn}\left(-\frac{{}^s\Gamma^n{}_{sq}}{2}\left(g^{rs}g_{uu,r} + 2g^{sm}E_{um}\right) + \left(g^{rn}g_{u[u,r}\right)_{,q]} - 2\left(g^{nm}E_{m[q}\right)_{,u]} \right) \\
&\quad + \Theta\left(g_{uu}\left(g_{u(p}g_{q)u,r} + g^{rr}g_{pq,r}\right) - E_{u(p}g_{q)u} - g^{rr}g_{u(p}g_{q)u,r} - 2g_{uu,(p}g_{q)u} \right) \\
&\quad - \Theta^2 g_{uu}\left(g_{up}g_{uq} + g^{rr}g_{pq}\right) \bigg),
\end{aligned} \tag{2.12}
$$

where C_{pqnm}, E_{pq}, E_{up} equations are given in appendix A. Some of these scalars can be obtained by contractions of the other scalars such as;

$$\Psi_{1T^i} = \Psi_{1^k{}_k{}^i}, \tag{2.13}$$

$$\Psi_{2S} = \Psi_{2T^k{}_k}, \tag{2.14}$$

$$\Psi_{3T^i} = \Psi_{3^k{}_k{}^i}, \tag{2.15}$$

and symmetric and antisymmetric part of the scalar $\Psi_{2T^{ij}}$ is obtained;

$$\Psi_{2T^{[ij]}} = \frac{1}{2}\Psi_{2^{ij}}, \tag{2.16}$$

$$\Psi_{2T^{(ij)}} = \frac{1}{2}\Psi_{2^{ikjk}}. \tag{2.17}$$

Analyzing of the classification for any dimension $D > 4$ is studied in [22]. According to study of [22], the irreducible components of these scalars are given;

$$\tilde{\Psi}_{1^{ijk}} \equiv \Psi_{1^{ijk}} - \frac{1}{D-3}\left(\delta_{ij}\Psi_{1T^k} - \delta_{ik}\Psi_{1T^j}\right), \tag{2.18}$$

$$\tilde{\Psi}_{2T^{(ij)}} \equiv \Psi_{2T^{(ij)}} - \frac{1}{D-2}\delta_{ij}\Psi_{2S}, \tag{2.19}$$

$$\begin{aligned}\tilde{\Psi}_{2^{ijk\ell}} \equiv \Psi_{2^{ijk\ell}} &- \frac{2}{D-4}\left(\delta_{ik}\tilde{\Psi}_{2T^{(j\ell)}} + \delta_{j\ell}\tilde{\Psi}_{2T^{(ik)}} - \delta_{i\ell}\tilde{\Psi}_{2T^{(jk)}} - \delta_{jk}\tilde{\Psi}_{2T^{(i\ell)}}\right)\\ &- \frac{2}{(D-2)(D-3)}\left(\delta_{ik}\delta_{j\ell} - \delta_{i\ell}\delta_{jk}\right)\Psi_{2S},\end{aligned} \tag{2.20}$$

$$\tilde{\Psi}_{3^{ijk}} \equiv \Psi_{3^{ijk}} - \frac{1}{D-3}\left(\delta_{ij}\Psi_{3T^k} - \delta_{ik}\Psi_{3T^j}\right), \tag{2.21}$$

which are dependent of the dimension of the spacetime. As we study the dimension becomes large, the second and third terms of the right side of the above equations are meaningless. Thus, we can conclude that, according to our study, the irreducible components of the Weyl salars can be written as,

$$\tilde{\Psi}_{1^{ijk}} \equiv \Psi_{1^{ijk}}, \qquad \tilde{\Psi}_{2T^{(ij)}} \equiv \Psi_{2T^{(ij)}}, \qquad \tilde{\Psi}_{2^{ijk\ell}} \equiv \Psi_{2^{ijk\ell}}, \qquad \tilde{\Psi}_{3^{ijk}} \equiv \Psi_{3^{ijk}}. \tag{2.22}$$

where the symmetric part of the $\Psi_{2T^{ij}}$ is equal to;

$$\begin{aligned}\Psi_{2T^{(ij)}} = m_i^p m_j^q \bigg(&\frac{1}{2}g_{pn}g^{ms}g_{us,r}\;{}^s\Gamma^n{}_{mq} + \frac{1}{4}g_{up,r}g_{uq,r} + \frac{1}{2}g_{pn}\left(g^{nm}g_{um,r}\right)_{,q} + g_{up}g_{uq}\left(\Theta^2 - \Theta_{,r}\right)\\ &-4g_{u(p}\Theta_{,q)} + \Theta g_{u(q}g_{p)u,r} + g_{u(p}g_{q)u,rr}\bigg).\end{aligned} \tag{2.23}$$

As the Weyl scalar $\Psi_{0^{ij}} = 0$, $\mathbf{k}$ is a primary WAND of the metric (2.1) and the spacetime is Type I. If Ψ_{1T^i} vanishes, $\mathbf{k}$ becomes multiple WAND since boost weight component of $+2, +1$ become zero. Meanwhile, vanishing the Weyl scalars determine the types of the spacetime: type I(a)-I(b), Type II(a)-II(b)-II(c)-II(d) Type N and Type O and their combinations such as TypeII(ac) or Type II(bcd), for primary WAND **k**. For instance, the spacetime is called Type II(c) when the Weyl scalars $\Psi_{0^{ij}} = \Psi_{1T^i} = \Psi_{1^{ijk}} = \Psi_{2^{ijkl}} = 0$ or Type III(a) when the scalars $\Psi_{0^{ij}} = \Psi_{1T^i} = \Psi_{1^{ijk}} = \Psi_{2S} = \Psi_{2T^{(ij)}} = \Psi_{2^{ijkl}} = \Psi_{2^{ij}} = \Psi_{3T^i} = 0$. Because of the components of the Weyl scalar $\Psi_{0^{ij}} = \Psi_{1^{ijk}} = 0$, the spacetime of shear-free, twist-free, expanding (or not expanding) can be classified as Type I(b) or more special. Additionally, metric (2.1) spacetime is not algebraically special, because of all +1 components of boost weight are not zero ($\Psi_{1T^i} \neq 0$).

Furthermore, the secondary WAND is the natural null vector ℓ which classifies the spacetime as the Type I_i - II_i - III_i and Type D, such as, the spacetime will be Type II_i, when $\Psi_{4^{ij}} = 0$ addition to $\Psi_{0^{ij}} = \Psi_{1T^i} = \Psi_{1^{ijk}} = 0$ or the spacetime becomes Type D(a) when the Weyl tensors $\Psi_{0^{ij}} = \Psi_{1T^i} = \Psi_{1^{ijk}} = \Psi_{2S} = \Psi_{3T^i} = \Psi_{3^{ijk}} = \Psi_{4^{ij}} = 0$. One prepared table 2 for the relationship between vanishing Weyl scalars and the spacetimes' classification, for both WANDs **k** and ℓ [22].

While the number of dimension goes to infinity, the classification of metric (2.1) did not change, the equations became only simpler than the functions, which were obtained for any dimensions $D > 4$. It is the power of the large D expansion method which allows to obtain analytical solutions for classification of this spacetime. Hereafter, we will determine (sub)-types of the higher dimensional, shear-free, twist-free and expanding or non-expanding spacetime for special conditions which will correspond specific spacetimes.

3 Kundt spacetime

Kundt spacetimes are shear-free, twist-free, non-expanding geometries which will be obtained by taking $\Theta = 0$ and the metric functions of spatial coordinates independently of parameter r at the metric (2.1). While the algebraic classification of the Kundt geometry for higher dimensions has been studied in [19, 22, 23], we will classify this geometry which the number of dimensions goes to infinity. Kundt spacetimes will be Type I(b) ($\Psi_{0^{ij}} = \Psi_{1^{ijk}} = 0$) or more special which is determined by the vanishing Weyl scalars. If we set the spacetime Type I(a) (it becomes Type II too), the Weyl scalar Ψ_{1T^i} have to vanish. As a result of it, the spacetime becomes algebraically special because all +1 component of the boost weight vanish and the metric function $g_{up,rr} = 0$ and it reads;

$$g_{up} = r d_p(u,x) + c_p(u,x).$$

If we keep going to determine the metric functions by vanishing Weyl scalars for classification, we get that from $\Psi_{2S} = 0$,

$$g_{uu} = \frac{r^2}{4} g^{pq}(u,x) d_p(u,x) d_q(u,x) + r l(u,x) + s(u,x),$$

where the solution is Type II(a) and the metric of Kundt spacetime becomes;

$$\begin{aligned} ds^2 &= g_{pq}\,(u,x)\, dx^p dx^q + 2\,(r d_p(u,x) + c_p(u,x))\, du dx^p - 2du dr \\ &\quad + \left(\frac{r^2}{4} g^{pq}(u,x) d_p(u,x) d_q(u,x) + r l(u,x) + s(u,x)\right) du^2. \end{aligned} \tag{3.1}$$

Components of Weyl Scalar	Types																
	WAND $\mathbf{k}$													WAND ℓ			
	I	I(a)	I(b)	II	II(a)	II(b)	II(c)	II(d)	III	III(a)	III(b)	N	O	I_i	II_i	III_i	D
$\Psi_{0^{ij}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ψ_{1T^i}		0		0	0	0	0	0	0	0	0	0	0		0	0	0
$\Psi_{1^{ijk}}$			0	0	0	0	0	0	0	0	0	0	0		0	0	0
Ψ_{2S}					0				0	0	0	0	0			0	
$\Psi_{2T^{(ij)}}$						0			0	0	0	0	0			0	
$\Psi_{2^{ijkl}}$							0		0	0	0	0	0			0	
$\Psi_{2^{ij}}$								0	0	0	0	0	0			0	
Ψ_{3T^i}										0		0	0				0
$\Psi_{3^{ijk}}$											0	0	0				0
$\Psi_{4^{ij}}$													0	0	0	0	0

Table 2. General algebraic classifications of shearfree, twistless spacetimes with necessary vanishing Weyl scalars for both WANDs $\mathbf{k}$ and ℓ.

Also, it easily becomes Type II(ad) by $\Psi_{2^{ij}} = 0$, which gives the result $d_p(u, x) = d_p(u)$. However, the other subtypes are not simply obtained by only vanishing components of Weyl scalar. Therefore, we will analyze several cases which is obtained by simplification of metric components.

3.1 All metric functions are independent of parameter r. (Corresponding pp-waves.)

pp-waves are one of the important subclasses of the Kundt spacetimes. They are defined in Brinkmann [19, 23, 28] form by the metric (3.1), as the all metric functions independent of the parameter r;

$$ds^2 = g_{pq}(u, x)\, dx^p dx^q + 2c_p(u, x) du dx^p - 2du dr + s(u, x) du^2. \tag{3.2}$$

With respect to independence of r, higher dimensional pp-waves can be classified as Type II(abd) ($\Psi_{0^{ij}} = \Psi_{1T^i} = \Psi_{1^{ijk}} = \Psi_{2S} = \Psi_{2T^{(ij)}} = \Psi_{2^{ij}} = 0$) or more special when the dimension number of the spacetime goes to infinity. Furthermore, if we set ${}^sR_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$ ($\Psi_{2^{ijkl}} = 0$) the pp-waves become Type III(a) and more special for multiple WAND $\mathbf{k}$. To make the pp-waves Type III(b) (simultaneously it becomes Type N), the condition of Weyl scalar $\Psi_{3^{ijk}} = 0$, gives that;

$$\left(g^{ns}E_{s[q}\right)_{,m]} = -g^{sk}E_{k[q}\ {}^s\Gamma^n_{m]s}.$$

There is one more condition which makes pp-waves Type O for WAND $\mathbf{k}$ is $\Psi_{4^{ij}} = 0$;

$$2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}E_{um}\ {}^s\Gamma^n_{sq}.$$

Moreover, only with this condition we can classify pp-waves Type I_i and II_i for secondary WAND ℓ. Higher dimensional pp-waves classification and obligatory conditions are summarized in table 3.

Types for WAND $\mathbf{k}$	Obligatory Conditions		Types for WAND ℓ
I	always	$2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	I_i
I(a)	always	$2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	$I(a)_i$
I(b)	always	$2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	$I(b)_i$
II	always	$2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	II_i
II(a)	always	$2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	$II(a)_i$
II(b)	always	$2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	$II(b)_i$
II(c)	${}^{s}R_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$	${}^{s}R_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$ $2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	$II(c)_i$
II(d)	always	$2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	$II(d)_i$
III	${}^{s}R_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$	${}^{s}R_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$ $2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	III_i
III(a)	${}^{s}R_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$	${}^{s}R_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$ $2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	$III(a)_i$
III(b)	${}^{s}R_{pqnm} = c_s(u,x)\ {}^{s}\Gamma^{s}{}_{p[n}g_{m]q}$, $\left(g^{ns}E_{s[q}\right)_{,m]} = -g^{sk}E_{k[q}\ {}^{s}\Gamma^{n}_{m]s}$	${}^{s}R_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$ $\left(g^{ns}E_{s[q}\right)_{,m]} = -g^{sk}E_{k[q}\ {}^{s}\Gamma^{n}_{m]s}$ $2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	$III(b)_i$
N	${}^{s}R_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$ $\left(g^{ns}E_{s[q}\right)_{,m]} = -g^{sk}E_{k[q}\ {}^{s}\Gamma^{n}_{m]s}$	$\left(g^{ns}E_{s[q}\right)_{,m]} = -g^{sk}E_{k[q}\ {}^{s}\Gamma^{n}_{m]s}$ $2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	D= (D(abd))
O	${}^{s}R_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$ $\left(g^{ns}E_{s[q}\right)_{,m]} = -g^{sk}E_{k[q}\ {}^{s}\Gamma^{n}_{m]s}$ $2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	${}^{s}R_{pqnm} = -g_{ps}\left(g^{rs}g_{q[n}\right)_{,m]}$ $\left(g^{ns}E_{s[q}\right)_{,m]} = -g^{sk}E_{k[q}\ {}^{s}\Gamma^{n}_{m]s}$, $2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}\ {}^{s}\Gamma^{n}{}_{sq}E_{um}$	D(c)

Table 3. Higher dimensional pp-waves classification with obligatory conditions for both WANDs $\mathbf{k}$ and ℓ as the dimension of the spacetime $D \to \infty$.

3.2 $\boldsymbol{g_{up} = 0, g_{uu} = rl(u,x)}$

According to this special case, the solution becomes Type II(abd) or more special because of $\Psi_{2T^{(ij)}} = \Psi_{2^{ij}} = 0$. Meanwhile, the spacetime will be Type II(c) as the Weyl scalar $\Psi_{2^{ijkl}} = 0$ which is provided by taking flat spatial spacetime ${}^{s}R_{pqnm} = 0$. Interestingly, when the metric function is $g_{uu,pr} = 0$ which is obtained by $\Psi_{3^{T^i}} = 0$ the spacetime becomes Type III(a). This can be summarized as;

$$l(u,x) \to l(u).$$

Moreover, this spacetime becomes Type III(b) (and also Type N), and Type O with respect to multiple WAND **k** if the Weyl scalars become zero as,

$$\Psi_{3^{ijk}} = 0 \rightarrow \left(g^{ns}E_{s[q}\right)_{,m]} = -g^{sk}E_{k[q}\ {}^{s}\Gamma^{n}_{m]s},$$
$$\Psi_{4^{ij}} = 0 \rightarrow 2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}E_{um}\ {}^{s}\Gamma^{n}_{sq}.$$

which are the same conditions of the all metric functions are independent of the parameter r case.

Additionally, when the metric functions are choosen $g_{up=0}$ and $g_{uu} = rl(u,x)$, algebraic classification for secondary WAND ℓ can be written as Type I_i and Type II_i if the condition $\Psi_{4^{ij}} = 0$ is satisfied which gives;

$$\Psi_{4^{ij}} = 0 \rightarrow 2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}E_{um}\ {}^{s}\Gamma^{n}_{sq}.$$

With this condition, if ${}^{s}R_{pqnm} = 0$ which means spatial spacetime is flat the classification becomes Type III_i for WAND ℓ. On the other hand, if we want to get Type D solutions, we have to set

$$\Psi_{3T^i} = 0 \rightarrow l(u,x) \rightarrow l(u),$$
$$\Psi_{3^{ijk}} = 0 \rightarrow \left(g^{ns}E_{s[q}\right)_{,m]} = -g^{sk}E_{k[q}\ {}^{s}\Gamma^{n}_{m]s},$$
$$\Psi_{4^{ij}} = 0 \rightarrow 2\left(g^{nm}E_{m[q}\right)_{,u]} = -g^{sm}E_{um}\ {}^{s}\Gamma^{n}_{sq}.$$

Also the spacetime becomes Type D(abd) with these conditions in this case.

4 Robinson-Trautman spacetime

Higher dimensional Robinson-Trautman geometry defines non-twist, shearfree spacetimes, which expand along the direction r in metric (2.1). General algebraic classification of RT in higher dimensions as dimension numbers go to infinity is given in table 2. In here we will discuss some special cases. First, Riemannian Type I and Ricci Type I will be studied which corresponds to expansion scalar $\Theta = \frac{1}{r}$ and $R_{rprq} = R_{rr} = 0$. The spatial metric of the spacetime becomes;

$$g_{pq} = r^2 h_{pq}(u,x).$$

According to this expansion scalar the spacetime is Type I and Type I(b) too, as $D \rightarrow \infty$. This spacetime becomes Type I(a) and also Type II and more special if the $\Psi_{1T^i} = 0$ which is obtained by;

$$g_{up} = r^2 d_p(u,x) + rc_p(u,x).$$

By using these metric functions, we can rewrite the non-vanishing Weyl scalars;

$$\Psi_{2S} = \frac{1}{2}g_{uu,rr} - \frac{g_{uu,r}}{r} + \frac{g_{uu}}{r^2} - \frac{g^{pq}}{4}(2rd_p + c_p)(2rd_q + c_q) + g^{rp}d_p, \tag{4.1}$$

$$\Psi_{2T^{(ij)}} = m_i^p m_j^q \left[\frac{r^2 h_{pn}}{2}\left[g^{ms}\ {}^{s}\Gamma^{n}{}_{mq}(2rd_s + c_s) + 2\left(g^{nm}(2rd_m + c_m)\right)_{,q}\right]\right.$$
$$\left. + 7r^2 d_p d_q + 5rd_p c_q + 5rd_q c_p + \frac{13}{4}c_p c_q\right], \tag{4.2}$$

$$\Psi_{2^{ijk\ell}} = m_i^p m_j^q m_k^n m_l^m \tilde{C}_{pqnm}, \tag{4.3}$$

$$\Psi_{2^{ij}} = \frac{1}{2} m_i^p m_j^q \left[(2rd_p + c_p)_{,q} - (2rd_q + c_q)_{,p} + \frac{2}{r} (E_{qm} - E_{pn}) \right], \tag{4.4}$$

$$\begin{aligned}
\Psi_{3T^i} = m_i^p \Bigg[& \frac{1}{2} d_p g_{uu} - \frac{1}{2} g_{uu,pr} - (rd_p + c_p) \left(\frac{r}{2} g_{uu,rr} - g_{uu,r} \right) + \frac{1}{2} (2rd_p + c_p)_{,u} \\
& - \frac{g_{uu}}{2r^2} \left(r^2 d_p + rc_p \right) - \frac{1}{2r} \left(g_{uu,p} + g^{rr} (2rd_p + c_p) + 2g^{rs} E_{sp} \right) \\
& + \frac{1}{2} g^{mn} E_{np} (2rd_m + c_m) \Bigg],
\end{aligned} \tag{4.5}$$

$$\begin{aligned}
\Psi_{3^{ijk}} = m_i^p m_j^q m_k^n \Bigg[& -\frac{1}{2} \left(r^2 d_p + rc_p \right) \left[(2rd_q + c_q)_{,m} - (2rd_m + c_m)_{,q} \right] \\
& - (rd_p + c_p)(E_{mn} - E_{qs}) + r^2 (d_m c_q - d_q c_m) \left(\frac{rd_p}{4} + (rd_p + c_p) \left(\frac{5}{4} + \frac{r}{2} \right) \right) \\
& + 3r (E_{pq} d_m - E_{pm} d_q) + \frac{5}{2} (E_{pq} c_m - E_{pm} c_q) - g^{rr} \Big[h_{pq} \left(r^2 d_m + rc_m \right) \\
& - h_{pm} \left(r^2 d_q + rc_q \right) \Big] + \frac{r^3}{2} \Big[rg^{\ell s}\, {}^s\Gamma^n{}_{\ell p} (rd_s + c_s) + \left(g^{n\ell} (2rd_\ell + c_\ell) \right)_{,p} \Big] \\
& \times \left[h_{mn} (rd_q + c_q) - h_{qn} (rd_m + c_m) \right] - \frac{r^2 h_{pn}}{2} \Big[\left(g^{rn} (2rd_m + c_m) \right)_{,q} \\
& - \left(g^{rn} (2rd_q + c_q) \right)_{,m} + g^{rs} \big({}^s\Gamma^n{}_{sq} (2rd_m + c_m) \\
& - {}^s\Gamma^n{}_{sm} (2rd_q + c_q) \big) + 4 \left(g^{ns} E_{s[q} \right)_{,m]} + 4g^{sk} E_{k[q}\, {}^s\Gamma^n{}_{m]s} \Big] \Bigg],
\end{aligned} \tag{4.6}$$

$$\begin{aligned}
\Psi_{4^{ij}} = m_i^p m_j^q \Bigg[& \frac{r}{2} (rd_q + c_q) \left(g_{uu,pr} - (2rd_p + c_p)_{,u} \right) + \frac{rg^{mn}}{2} \left(E_{np} (rd_q + c_q) + E_{nq} (rd_p + c_p) \right) \\
& + \frac{g_{uu}}{4} \Big[r^2 g^{ms} (2rd_s + c_s) \left[h_{qn}\, {}^s\Gamma^n{}_{mp} + h_{pn}\, {}^s\Gamma^n{}_{mq} \right] + r^2 \big[h_{qn} \left(g^{nm} (2rd_m + c_m) \right)_{,p} \\
& + h_{pn} \left(g^{nm} (2rd_m + c_m) \right)_{,q} \big] + 2 \left(2r^2 d_p d_q + rd_p c_q + rd_q c_p \right) + 4g^{rr} h_{pq} - \frac{8}{r} E_{(pq)} \Big] \\
& - \frac{rg^{rm}}{4} (2rd_m + c_m) \left(4r^2 d_p d_q + 3rd_p c_q + 3rd_q c_p + 2c_p c_q \right) - g_{uu,p} (rd_q + c_q) \\
& + g_{uu,q} (rd_p + c_p) + \left(r^2 d_p + rc_p \right) \left(r^2 d_q + rc_q \right) \left[\frac{g^{pq}}{2} (2rd_p + c_p)(2rd_q + c_q) + \frac{g_{uu,r}}{r} \right] \\
& - \frac{g^{rr}}{2} \left[(rd_p + c_p)(2rd_q + c_q) + (rd_q + c_q)(2rd_p + c_p) \right] \\
& - r^2 h_{pn} \left(- \frac{{}^s\Gamma^n{}_{sq}}{2} \left(g^{rs} g_{uu,r} + 2g^{sm} E_{um} \right) + \left(g^{rn} g_{u[u,r} \right)_{,q]} - 2 \left(g^{nm} E_{m[q} \right)_{,u]} \right) \Bigg],
\end{aligned} \tag{4.7}$$

where $\tilde{C}_{pqnm}$ is the Weyl tensor for the obtained metric functions. We can conclude that, RT spacetime algebraic classification is not revealed analytically for this expansion scalar and these metric functions, as $D \to \infty$.

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On the other hand, we can determine the classification of RT spacetime with the same expansion scalar $\Theta = \frac{1}{r}$ for off-diagonal terms (g_{up}) vanish. According to this simplification, $d_p = c_p = 0$ and the metric coefficients become $g^{rp} = 0$ and $g^{rr} = -g_{uu}$, thus, the metric (2.1) becomes,

$$ds^2 = r^2 h_{pq}(u,x)dx^p dx^q - 2dudr + g_{uu}(u,r,x)du^2. \tag{4.8}$$

This spacetime is Type II(b) or more special as the Weyl scalars $\Psi_{0^{ij}} = \Psi_{1T^i} = \Psi_{1^{ijk}} = \Psi_{2T^{(ij)}} = 0$ and it is algebraically special. Additionally, it will be Type II(ab) or more special for the limit of $D \to \infty$ when the Weyl scalar $\Psi_{2S} = 0$ which gives,

$$g_{uu} = c_1(u,x)r^2 - 2c_2(u,x)r. \tag{4.9}$$

RT spacetime will be Type II(bc) or more special as the Weyl scalar $\Psi_{2^{ijk\ell}}$ vanishes which reads;

$${}^sR_{pqmn} = r^3 h_{q[m}h_{n]p,u} + 2r^2 g^{rr} h_{p[n}h_{m]q}. \tag{4.10}$$

When we set $\Psi_{2^{ij}} = 0$, the spacetime will be Type II(d) or more special and we get $h_{qm,u} = h_{pn,u}$. Without loss of generality we can choose the metric coefficient $h_{pq}(u,x) \to h_{pq}(x)$ for the Type II(abcd) (or Type III) with these results. RT will be Type III(a) with the condition of $\Psi_{3T^i} = 0 \to (rg_{uu,p})_{,r} = 0 \to c_1(u,x) \to c_1(u), c_2(u,x) \to c_2(u)$. On the other hand, the spacetime becomes Type III(b) if the metric coefficients satisfy the condition that;

$$g^{ns}{}_{[,m}h_{q]s,u} = g^{sk}\ {}^s\Gamma^n{}_{s[q}h_{m]k,u}. \tag{4.11}$$

It will be Type O when above conditions are satisfied with $\Psi_{4^{ij}} = 0$ which gives $g_{uu}^2 h_{pq} = 0$. This is an unphysical result for our purpose because the metric functions g_{uu} or h_{pq} vanishes which is changed the spacetime geometry.

Also, we can introduce classification for the secondary WAND ℓ. RT spacetime will be Type I_i and II_i only the component of Weyl scalar $\Psi_{4^{ij}}$ vanishes and it yields;

$$rh_{pq,u} + g_{uu}^2 h_{pq} + \frac{r^2 h_p n}{2}\left[g^{sm}\ {}^s\Gamma^n{}_{sq} g_{uu,m} + (g^{nm}g_{uu,m})_{,q} - (g^{nm}h_{mq,u})_{u}\right] = 0. \tag{4.12}$$

If the metric functions satisfy equations (4.9), (4.10), (4.11) and $h_{pq}(u,x) \to h_{pq}(u)$ with equation (4.12) RT spacetime is Type III_i for secondary WAND ℓ.

5 Conclusion

Algebraic classification of the higher dimensional RT and Kundt spacetimes were investigated with the method of taking the limit of dimension of spacetime $D \to \infty$. The results supported the method that the limit of dimension goes to infinity which simplify the theory of general relativity. In general, nor at this limit or any dimension $D > 4$, without any restrictions, classification of RT spacetime which is Type I(b) remains unchanged. Also the spacetime is not algebraically special and the primary WAND **k** is a WAND because all +1 components of boost weight do not vanish.

As Kundt spacetime corresponds $\Theta = 0$ condition of RT spacetime, in general, at least it is Type I(b). We obtained other types and subtypes by setting the components of Weyl tensor are zero. When the all metric functions are chosen independently of parameter r, which correspond to pp-waves, the spacetime became Type II(abd). Restrictions and the matching types and subtypes are summarized at the table 3 for this case.

Although, general classification of RT spacetime with necessary vanishing Weyl scalars for both WANDs was given at table 2, several different cases were discussed at the last section. When we set the $\Theta = \frac{1}{r}$ and off-diagonal terms $g_{up} = 0$, the spacetime became algebraically special and it is Type II(b) or more special.

This paper was prepared to fill the gap in the literature by analyzing the algebraic classification of RT spacetime with the limit of dimension $D \to 0$. In the future, the special types and subtypes can be studied by solving field equations which helps to understand the limitation method.

A Curvature tensors of general RT spacetime

The non-zero Christoffel symbols of the metric (2.1) are obtained;

$$\Gamma^{u}{}_{uu} = \frac{1}{2} g_{uu,r}, \tag{A.1}$$

$$\Gamma^{u}{}_{up} = \frac{1}{2} g_{up,r}, \tag{A.2}$$

$$\Gamma^{u}{}_{pq} = \Theta g_{pq}, \tag{A.3}$$

$$\Gamma^{r}{}_{ur} = \frac{1}{2}\left(g^{rp} g_{up,r} - g_{uu,r}\right), \tag{A.4}$$

$$\Gamma^{r}{}_{up} = \frac{1}{2}\left(-g^{rr} g_{up,r} - g_{uu,p} + 2g^{rn} E_{np}\right), \tag{A.5}$$

$$\Gamma^{r}{}_{uu} = \frac{1}{2}\left(-g^{rr} g_{uu,r} - g_{uu,u} + 2g^{rn} E_{un}\right), \tag{A.6}$$

$$\Gamma^{r}{}_{pq} = -\Theta g^{rr} g_{pq} + \frac{1}{2} g_{pq,u} - g_{u(p,q)} + g_{un}\ {}^{s}\Gamma^{n}{}_{pq}, \tag{A.7}$$

$$\Gamma^{r}{}_{rp} = \frac{1}{2}\left(2\Theta g_{up} - g_{up,r}\right), \tag{A.8}$$

$$\Gamma^{p}{}_{uu} = \frac{1}{2}\left(-g^{rp} g_{uu,r} + 2g^{pn} E_{un}\right), \tag{A.9}$$

$$\Gamma^{p}{}_{ur} = \frac{1}{2} g^{pq} g_{uq,r}, \tag{A.10}$$

$$\Gamma^{p}{}_{uq} = \frac{1}{2}\left(-g^{rp} g_{uq,r} + 2g_{pn} E_{nq}\right), \tag{A.11}$$

$$\Gamma^{p}{}_{rq} = \Theta \delta^{p}{}_{q}, \tag{A.12}$$

$$\Gamma^{m}{}_{pq} = -\Theta g_{pq} g^{rm} + {}^{s}\Gamma^{m}{}_{pq}, \tag{A.13}$$

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where ${}^s\Gamma^n{}_{pq}$ is the Chrisstoffel symbol of the spatial metric g_{pq} and,

$$E_{pq} = g_{u[p,q]} + \frac{1}{2}g_{pq,u}, \tag{A.14}$$
$$E_{up} = g_{u[p,u]} + \frac{1}{2}g_{up,u}. \tag{A.15}$$

The Rieman tensors of the metric (2.1) are;

$$R_{prrq} = g_{pq}\left(\Theta_{,r} + \Theta^2\right), \tag{A.16}$$
$$R_{ruur} = \frac{1}{2}g_{uu,rr} - \frac{1}{4}g^{pq}g_{up,r}g_{uq,r}, \tag{A.17}$$
$$R_{ruup} = g_{u[u,p]r} + \frac{1}{2}\Theta\left(2E_{up} - g_{up}g_{uu,r}\right) - \frac{1}{2}g^{mn}g_{um,r}E_{np} + \frac{1}{4}g^{rm}g_{um,r}g_{up,r}, \tag{A.18}$$
$$R_{rurp} = -\frac{1}{2}g_{up,rr} + \frac{1}{2}\Theta g_{up,r}, \tag{A.19}$$
$$R_{rupq} = g_{u[p,q]r} + \Theta\left(g_{u[p}g_{q]u,r} + E_{qm} - E_{pn}\right), \tag{A.20}$$
$$R_{prmq} = 2\Theta^2 g_{p[m}g_{q]u} + \Theta g_{p[q}g_{m]u,r} + 2g_{p[q}\Theta_{,m]}, \tag{A.21}$$
$$R_{pruq} = -\frac{1}{2}g_{pn}g^{ms}g_{us,r}\ {}^s\Gamma^n{}_{mq} - \frac{1}{4}g_{up,r}g_{uq,r} - \frac{1}{2}g_{pn}\left(g^{nm}g_{um,r}\right)_{,q} + g_{pq}\Theta_{,u} + \frac{\Theta}{2}\left(-g_{pq}g^{rm}g_{um,r} + g_{pq}g_{uu,r} + g_{uq}g_{up,r} + 2E_{qp}\right), \tag{A.22}$$
$$R_{pumq} = g_{up}g_{u[q,m]r} + E_{p[m}g_{q]u,r} + g_{pn}g^{rs}\ {}^s\Gamma^n{}_{s[q}g_{m]u,r} + 2g_{pn}g^{sk}E_{k[q}\ {}^s\Gamma^n{}_{m]s} + g_{pn}\left(g^{rn}g_{u[m,r}\right)_{,q]} + 2g_{pn}\left(g^{ns}E_{s[q}\right)_{,m]} + \Theta\left[g_{uu,[m}g_{q]p} + g^{rr}g_{p[q}g_{m]u,r} + 2g^{rs}E_{s[q}g_{m]p}\right], \tag{A.23}$$
$$R_{puqu} = g_{up}g_{u[u,q]r} - E_{p[u}g_{q]u,r} + \frac{1}{2}g^{rs}g_{uq,r}E_{ps} - g^{s\ell}E_{ps}E_{\ell q} - g_{pn}\left(g^{rn}g_{u[u,r}\right)_{,q]} - \frac{1}{2}g_{pn}\ {}^s\Gamma^n{}_{sq}\left(g^{rs}g_{uu,r} + 2g^{sm}E_{um}\right) + \frac{1}{4}g_{up,r}\left(g_{uu,q} + g^{rr}g_{uq,r} - 2g^{rs}E_{sq}\right) - 2g_{pn}\left(g^{nm}E_{m[q}\right)_{,u]} + \frac{1}{2}\Theta g_{pq}\left(-g_{uu,u} - g^{rr}g_{uu,r} + 2g^{rs}E_{us}\right), \tag{A.24}$$
$$R_{pqmn} = {}^sR_{pqmn} + \frac{1}{2}g_{up}g_{q[n,r}g_{m]u,r} + g_{up}g_{q[n,m]r} + g_{ps}\left(g^{rs}g_{q[m}\right)_{,n]} + g_{ps}g^{r\ell}\ {}^s\Gamma^s_{\ell[n}g_{m]q,r} + \Theta\Big[2g_{p[m}\tilde{E}_{n]q} + g^{rr}g_{p[n}g_{m]q,r} + 2E_{p[m}g_{n]q} - g^{rk}\left[g_{kq,[m}g_{n]p} + g_{p[n}g_{m]k,q} - g_{p[n}g_{m]q,k}\right] + 2g_{up}g_{q[m}g_{n]u,r}\Big], \tag{A.25}$$

where $\tilde{E}_{pq} = g_{u(p,q)} - \frac{1}{2}g_{pq,u}$.

Ricci tensors become;

$$R_{rr}=-(D-2)\left(\Theta^2+\Theta_{,r}\right), \tag{A.26}$$

$$R_{rp}=-\frac{1}{2}g_{up,rr}+\Theta_{,r}g_{up}+(D-2)\Theta^2 g_{up}-(D-3)\Theta_{,p}-\frac{D-4}{2}\Theta g_{up,r}, \tag{A.27}$$

$$\begin{aligned}R_{ru}=&-\frac{1}{2}g_{uu,rr}+\frac{1}{2}g^{rp}g_{up,rr}+\frac{1}{2}g^{ms}g_{us,r}\ {}^{s}\Gamma^{q}{}_{mq}+\frac{1}{2}\left(g^{mq}g_{um,r}\right)_{,q}-g^{pq}\Theta E_{qp}\\&+\frac{D-4}{2}\Theta g^{rp}g_{up,r}-\frac{D-2}{2}\Theta g_{uu,r}-(D-2)\Theta_{,u},\end{aligned} \tag{A.28}$$

$$\begin{aligned}R_{uu}=&-\frac{1}{2}g^{rr}g_{uu,rr}-\frac{1}{4}g^{rm}g^{rp}g_{um,r}g_{up,r}-\left(g^{rp}g_{u[u,r}\right)_{,q]}+\frac{1}{4}g^{pq}g_{up,r}g_{uu,q}-2\left(g^{mq}E_{m[q}\right)_{,u]}\\&+\frac{1}{2}g^{rp}g^{mn}g_{um,r}E_{np}-g^{pq}E_{p[u}g_{q]u,r}-g^{pq}g^{s\ell}E_{ps}E_{\ell q}-\frac{1}{2}\ {}^{s}\Gamma^{q}{}_{sq}\left(g^{rs}+2g^{sm}E_{um}\right)\\&+\frac{1}{2}\Theta g^{rp}g_{up}g_{uu,r}+\frac{1}{2}\Theta\left(-(D-2)\left(g_{uu,u}+g^{rr}g_{uu,r}\right)+2(D-3)g^{rp}E_{up}\right),\end{aligned} \tag{A.29}$$

$$\begin{aligned}R_{up}=&-\frac{1}{2}g^{rr}g_{up,rr}-g_{u[u,p]r}+\left(g^{rm}g_{u[m,r}\right)_{,p]}-\frac{1}{2}g^{rm}g_{um,r}g_{up,r}+\frac{1}{2}g^{mq}E_{qm}g_{up,r}\\&+g^{rs}\ {}^{s}\Gamma^{m}{}_{s[p}g_{m]u,r}-\frac{1}{2}g^{rq}g_{pn}g^{ms}g_{us,r}\ {}^{s}\Gamma^{n}{}_{mq}+2g^{sk}E_{k[p}\ {}^{s}\Gamma^{m}{}_{m]s}+2\left(g^{ms}E_{s}[p\right)_{,m]}\\&-\frac{1}{2}g^{rq}g_{pn}\left(g^{nm}g_{um,r}\right)_{,q}+g_{up}\Theta_{,u}-\Theta\left(E_{up}-g_{up}g_{uu,r}+g^{rp}E_{qn}\right)\\&+\frac{\Theta}{2}\left(-(D-3)g_{uu,p}-(D-3)g^{rr}g_{up,r}+2Dg^{rs}E_{sp}\right),\end{aligned} \tag{A.30}$$

$$\begin{aligned}R_{pq}=&\ {}^{s}R_{pq}-\frac{1}{2}g_{up,r}g_{uq,r}-\left(g^{nm}g_{um,r}\right)_{(,q}g_{p)n}+g^{r\ell}\ {}^{s}\Gamma^{n}{}_{\ell[q}g_{n]p,r}-g^{ms}g_{us,r}\ {}^{s}\Gamma^{n}{}_{m(p}g_{q)n}\\&+g^{rn}g_{p[q,n]r}+\left(g^{rn}g_{p[n}\right)_{,q]}-2\Theta^2 g_{up}g_{uq}+2g_{u(p}\Theta_{,q)}+\Theta g^{rn}g_{p[n}g_{q]u,r}\\&+g_{pq}\left(2\Theta_{,u}+\Theta g_{uu,r}-g^{rr}\Theta_{,r}+\Theta^2 g_{uu}+g^{mn}g_{um}g_{un}-2g^{rm}\Theta_{,m}+\Theta g^{mn}E_{mn}\right)\\&+\Theta(D-3)\left(\tilde{E}_{qp}-\Theta g^{rr}g_{pq}+g^{rk}g_{k(p,q)}-\frac{g^{rk}g_{pq,k}}{2}\right),\end{aligned} \tag{A.31}$$

Ricci scalar becomes;

$$\begin{aligned}R=&\ {}^{s}R+g_{uu,rr}-2g^{rp}g_{up,rr}-g^{ms}g_{us,r}\ {}^{s}\Gamma^{q}{}_{mq}-\left(g^{mq}g_{um,r}\right)_{,q}+2g^{rp}g_{up}\Theta_{,r}\\&+g^{pq}\left[-\frac{1}{2}g_{up,r}g_{uq,r}+g^{r\ell}\ {}^{s}\Gamma^{n}{}_{\ell[q}g_{n]p,r}+g^{rn}g_{p[q,n]r}+\left(g^{rn}g_{p[n}\right)_{,q]}-2\Theta^2 g_{up}g_{uq}\right]\\&+\frac{5D-19}{2}\Theta g^{rp}g_{up,r}+2(D-2)\left(\Theta g_{uu,r}+2\Theta_{,u}-g^{rr}\Theta_{,r}+g_{uu}\Theta^2\right)\\&+(D-2)\Theta^2 g^{rp}g_{up}-4(D-3)g^{rp}\Theta_{,p}+D\Theta g^{pq}E_{qp}+(D-2)g^{pq}g_{up}g_{un}\\&+\Theta(D-3)g^{pq}\left(\tilde{E}_{qp}+g^{rk}g_{k(p,q)}-\frac{g^{rk}g_{pq,k}}{2}\right)-\Theta^2(D-2)(D-3)g^{rr}\end{aligned} \tag{A.32}$$

As dimension $D \to \infty$ the Weyl tensor of the metric (2.1) becomes;

$$C_{rprq} = 0, \tag{A.33}$$

$$C_{rpru} = -\frac{1}{2} g_{up,rr} + \Theta g_{up,r} + \Theta_{,p} + g_{up}\Theta_{,r}\,, \tag{A.34}$$

$$C_{prmq} = 0, \tag{A.35}$$

$$C_{ruru} = -\left(\frac{1}{2} g_{uu,r} - \Theta g_{uu}\right)_{,r} + \frac{1}{4} g^{pq} g_{up,r} g_{uq,r} + 2\Theta_{,u} - \Theta g^{rp} g_{up,r} + \Theta^2 g^{rp} g_{up}, \tag{A.36}$$

$$\begin{aligned} C_{rpuq} = {} & \frac{1}{2} g_{pn} g^{ms} g_{us,r}\, {}^s\Gamma^n{}_{mq} + \frac{1}{4} g_{up,r} g_{uq,r} + \frac{1}{2} g_{pn}\left(g^{nm} g_{um,r}\right)_{,q} + \Theta^2\left(g_{up} g_{uq} + g^{rr} g_{pq}\right) \\ & - g_{up}\Theta_{,q} - \Theta\left(2E_{qp} - g_{u(p} g_{q)u,r}\right), \end{aligned} \tag{A.37}$$

$$C_{rupq} = g_{u[p,q]r} - 2 g_{u[p}\Theta_{,q]} + \Theta\left(E_{qm} - E_{pn}\right), \tag{A.38}$$

$$\begin{aligned} C_{pqmn} = {} & {}^sR_{pqmn} + \frac{1}{2} g_{up} g_{q[n,r} g_{m]u,r} + g_{up} g_{q[n,m]r} + g_{ps}\left(g^{rs} g_{q[m}\right)_{,n]} + 2\Theta E_{p[m} g_{n]q} \\ & + g_{ps} g^{r\ell}\, {}^s\Gamma^s{}_{\ell[n} g_{m]q,r} + 2\Theta g_{up} g_{q[m} g_{n]u,r} - 2\Theta^2 g^{rr} g_{p[m} g_{n]q}, \end{aligned} \tag{A.39}$$

$$\begin{aligned} C_{ruup} = {} & g_{u[u,p]r} - \frac{1}{2} g^{mn} g_{um,r} E_{np} + \frac{1}{4} g^{rm} g_{um,r} g_{up,r} + 2 g_{u[p}\Theta_{,u]} + \Theta^2 g_{up} g^{rn} g_{un} \\ & + \frac{\Theta}{2}\left(2E_{up} - g_{uu} g_{up,r} - 2 g_{up} g^{rq} g_{uq,r} + g^{rr} g_{up,r} + g_{uu,p} + 2 g^{rs} E_{sp}\right), \end{aligned} \tag{A.40}$$

$$\begin{aligned} C_{upmq} = {} & -g_{up} g_{u[q,m]r} - g_{pn}\left(g^{rn} g_{u[m,r}\right)_{,q]} - E_{p[m} g_{q]u,r} - 2 g_{pn}\left(g^{ns} E_{s[q}\right)_{,m]} \\ & - g_{pn} g^{rs}\, {}^s\Gamma^n{}_{s[q} g_{m]u,r} - 2 g_{pn} g^{sk} E_{k[q}\, {}^s\Gamma^n{}_{m]s} - 2\Theta^2 g^{rr} g_{u[m} g_{q]p}, \end{aligned} \tag{A.41}$$

$$\begin{aligned} C_{upuq} = {} & g_{up} g_{u[u,q]r} - g_{pn}\left(\left(g^{rn} g_{u[u,r]}\right)_{,q]} - 2\left(g^{nm} E_{m[q}\right)_{,u]} - \frac{{}^s\Gamma^n{}_{sq}}{2}\left(g^{rs} g_{uu,r} + 2 g^{sm} E_{um}\right)\right) \\ & + \frac{1}{4} g^{rr} g_{up,r} g_{uq,r} + \Theta\left(g^{rr} g_{uu} g_{pq,r} - g^{rr} g_{u(p} g_{q)u,r} - g_{uu,(p} g_{g)u} + 2 g^{rs} E_{s(p} g_{q)u}\right) \\ & - 2\Theta^2 g^{rr} g_{u[u} g_{p]q}. \end{aligned} \tag{A.42}$$

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